4.5. How does the derivative affect the shape of a graph

Thursday, August 3, 2017 7:31 AM

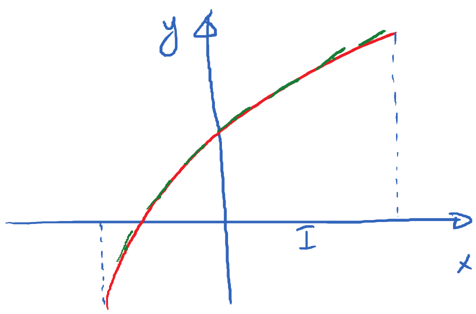
Goals: (1) Use the first derivative test to determine the intervals of increasing / decreasing and the local extrema of a function.

(2) Use the second derivative test to determine the intervals of concavity of a function and find inflection points

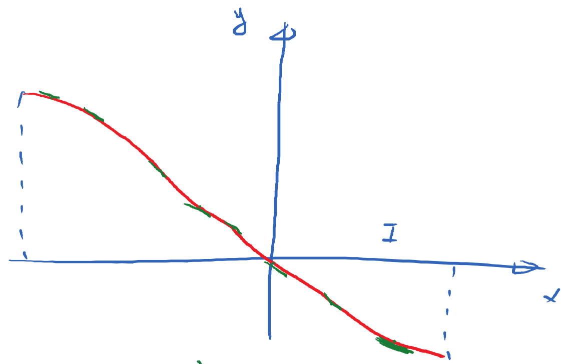
(1) What does f' tell you about the graph f ?

(I) If $f'(x) > 0$ on an interval I, then f is increasing on I.

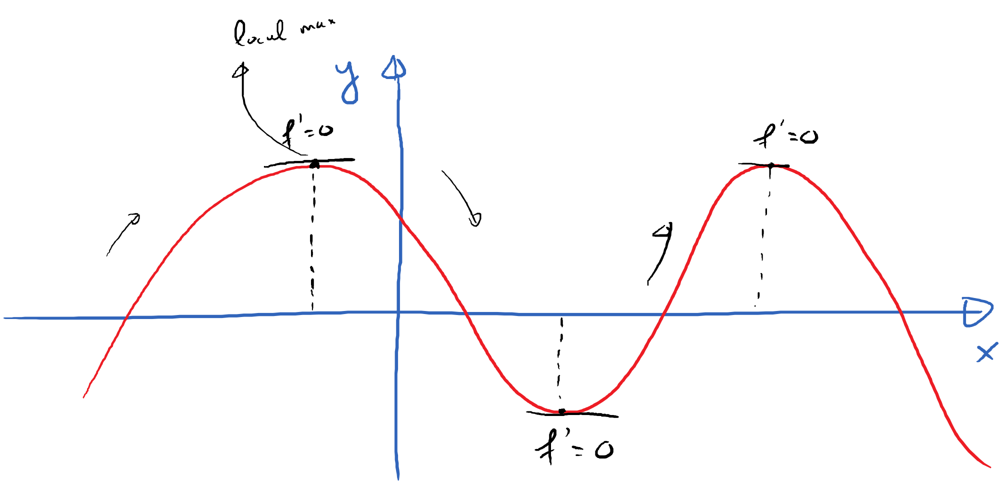
(II) If $f'(x) < 0$ on an interval I, then f is decreasing on I



$f' > 0$



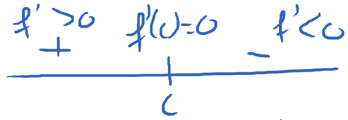
$f' < 0$



The first derivative test

If $x = c$ is a critical point of a function f $\Big\langle$ either $f'(c) = 0$ or $f'(c)$ is undefined, then

(1) f has a local max at $x = c$ if f' change sign from positive to negative at c



or $f'(c)$ undefined

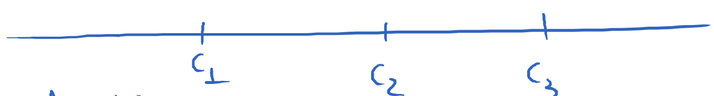
(2) f has a local min at $x = c$ if f' changes sign from negative to positive at c.

(3) f has neither a local max nor local min if f' doesn't change sign at c.

How to use this test to determine intervals of increasing / decreasing and find local max / min

Step 1: Find f'

Step 2: Use f' to find all critical points.



Step 3: Consider the sign of f' on these subintervals and draw conclusion.

E.g. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

Q: Find intervals of increasing and decreasing of f.
Find local max / local min of f.

Step 1: $f'(x) = 12x^3 - 12x^2 - 24x$

Step 2: Find critical points: $f'(x)$ is always defined

$$f'(x) = 0.$$

$$12x^3 - 12x^2 - 24x = 0$$

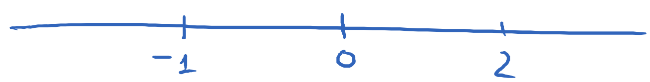
$$12x(x^2 - x - 2) = 0$$

$$\boxed{12x(x-2)(x+1)} = 0$$

$$x = 0 \; ; \quad x = 2 \; ; \quad x = -1$$

Critical points: $-1, 0, 2$

Step 3:



Make a table to consider the sign of f'

x	$-\infty$		$\boxed{-1}$		0		2		∞
$f'(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		$\searrow$	local min @ $x=-1$	$\nearrow$	local max @ $x=0$	$\searrow$	local min @ $x=2$	$\nearrow$	

Step 4: Conclusion:

Interval of increasing: $(-1, 0) \cup (2, \infty)$

decreasing : $(-\infty,-1)\cup(0,2)$

local min : at $x=-1$ and $x=2$.

local max at $x=0$

To find value of local min / max plug these x into original function.

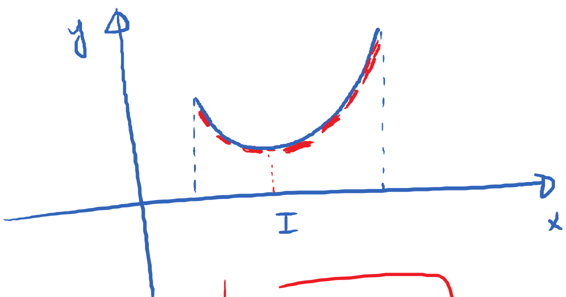
Ex. Use 1^{st} derivative test to find interval of increasing / decreasing and local max / min of

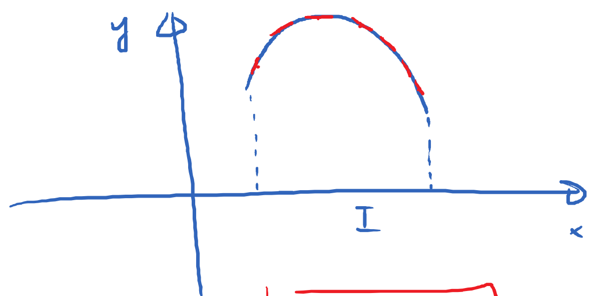
$$f(x)=2\ln(x)-5\arctan(x)$$

Domain : $(0,\infty)$

Solved in class

What does f'' tell you about the graph of f ?





concave up

Change in 1^{st} derivative

f' negative $\to$ f' less negative $\to$ $f'=0$

$\to$ f' positive $\to$ f' more ~~negative~~ positive

$\leftrightarrow$ f' increases $\leftrightarrow$ $\boxed{f''>0}$

concave down

$\boxed{f''<0}$

sad, negative $f''<0$

positive $f''>0$