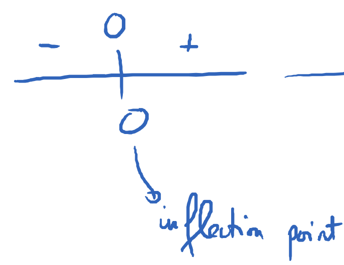
Concavity Test:

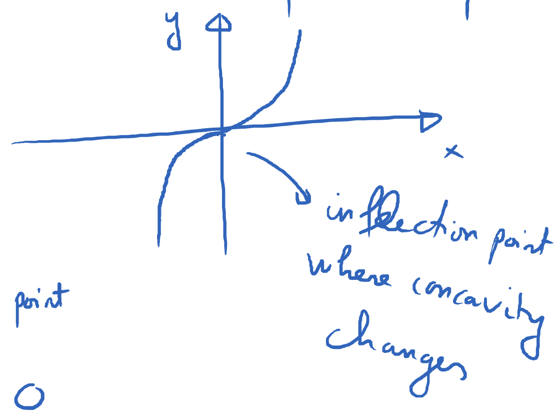
(I) If $f''(x) > 0$ on an interval I, then f is concave up on I.

(II) If $f''(x) < 0$ on an interval I, then f is concave down on I.

Inflection Point.

If $f''(c) = 0$ or $f''(c)$ is undefined and f'' changes signs at c, then we say that c is an inflection point of the function

f'' 



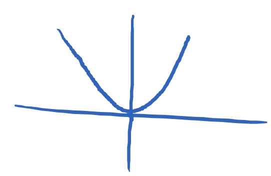
$f(x) = x^3$.

$f'(x) = 3x^2$

$f''(x) = \boxed{6x} = 0$ when $x = 0$

E.g. $f(x) = x^2$.

$f'(x) = 2x$; $f''(x) = 2$



E.g. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

$f'(x) = 12x^3 - 12x^2 - 24x$

Q: Determine the interval of concavity and inflection points of f.

$f''(x) = 36x^2 - 24x - 24$

$$f''(x) = 0$$

$$36x^2 - 24x - 24 = 0$$

$$12(3x^2 - 2x - 2) = 0$$

$$3x^2 - 2x - 2 = 0$$

zeros: $\dfrac{2 \pm \sqrt{4 + 24}}{6} = \dfrac{2 \pm \sqrt{28}}{6}$

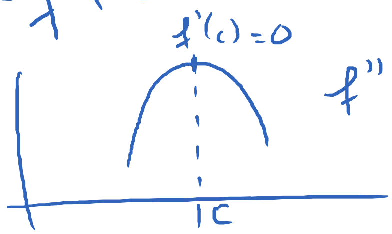
x		$\frac{2-\sqrt{28}}{6}$		$\frac{2+\sqrt{28}}{6}$	
$f''(x)$	$+$	0	$-$	0	$+$
$f(x)$					
		inflection		inflection	

Concave up: $\left(-\infty, \dfrac{2-\sqrt{28}}{6}\right) \cup \left(\dfrac{2+\sqrt{28}}{6}, \infty\right)$

concave down: $\left(\dfrac{2-\sqrt{28}}{6}, \dfrac{2+\sqrt{28}}{6}\right)$

The second derivative can also help us to classify local max / local min in many cases.

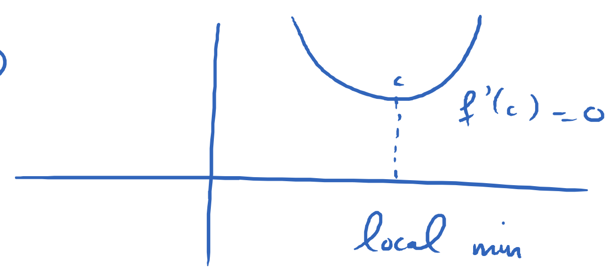
$f'(c) = 0 \Longleftrightarrow c \longrightarrow$ critical #



$f''(c) < 0$

local max

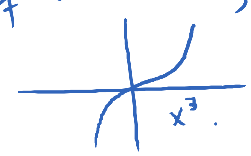
$f''(c) > 0$



local min

* If $f'(c) = 0$ and $f''(c) < 0$, then c corresponds to a local max

* If $f'(c)=0$ and $f''(c) > 0$, then c corresponds to a local min
* If $f'(c)=0$; $f''(c)=0 \longrightarrow$ inconclusive.



Solved # 5 Review 3.