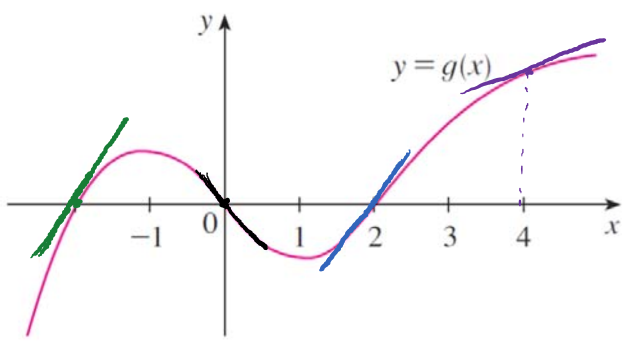
3.2. The Derivative as a function.

$\boxed{0}$ $g'(-2)$ $g'(0)$ $g'(2)$ $g'(4)$

$g'(0) < 0 < g'(4)$

$< g'(2) < g'(-2)$



E.g. where derivative DNE at a point.

$$f(x) = \begin{cases} 2 & \text{if } x < 2 \\ x & \text{if } x \geq 2. \end{cases}$$

Show that $f'(2)$ DNE?

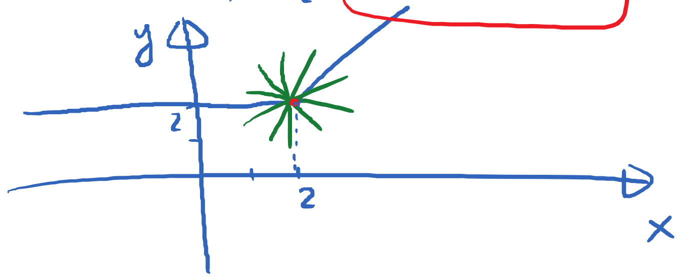
$$f'(2) = \boxed{\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}}$$

$$\lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{x-2}{x-2} = \lim_{x\to 2}(1) = \boxed{1}$$

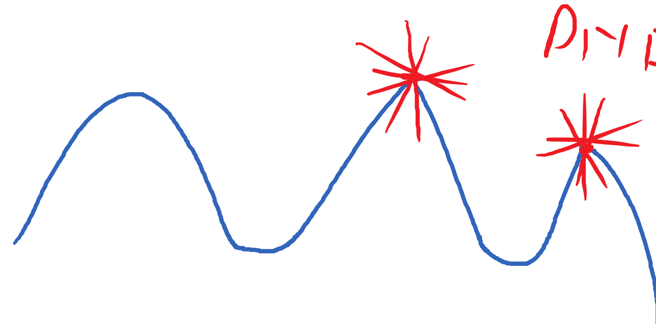
$$\lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x\to 2} \frac{2-2}{x-2} =$$

$$= \lim_{x\to 2}(0) = \boxed{0}$$

Hence, $\lim_{x\to 2} \boxed{\frac{f(x)-f(2)}{x-2}}$ DNE. Hence, $f'(2)$ DNE.



Note: corner $\longrightarrow$ derivative DNE.



Derivative as a function.

$f(x) = x^2$. $\quad f'(1) = \lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = (x+h)^2 . \qquad f(x) = x^2$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{- x^2}}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (2x + h) = 2x$$

$$f'(x) = 2x$$

$f'(10000)$

$= 20000$

$f'(\frac{1}{2}) = 1$

In most situation, it is much more useful to have a formula for the derivative at any arbitrary point x.

All we need to do is to replace a by x in the second formula in the definition for the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(provided that limit exists)

This defines a function $y = f'(x)$ which is called the derivative function of f, in short, the derivative of f.

E.g. $f(x) = \sqrt{x-7}$

$$f'(x) = \frac{1}{2\sqrt{x-7}}$$

$$f'(14) = \frac{1}{2\sqrt{7}}$$

$$f'(21) = \frac{1}{2\sqrt{14}}.$$

Def: We say that f is differentiable at a point x if $f'(x)$ exists.

E.g. $f(x) = x^2$

$$f'(x) = 2x.$$

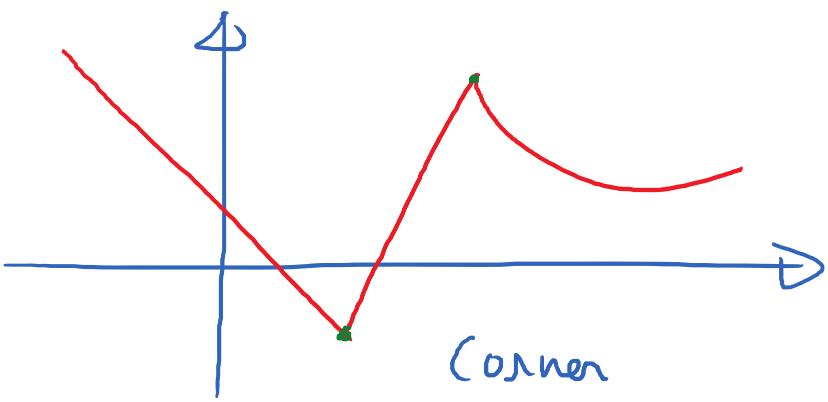
f is differentiable everywhere (on $(-\infty, \infty)$)

$f(x) = \sqrt{x-7}$. $\quad f'(x) = \dfrac{1}{2\sqrt{x-7}}$.

f is differentiable on $(7, \infty)$

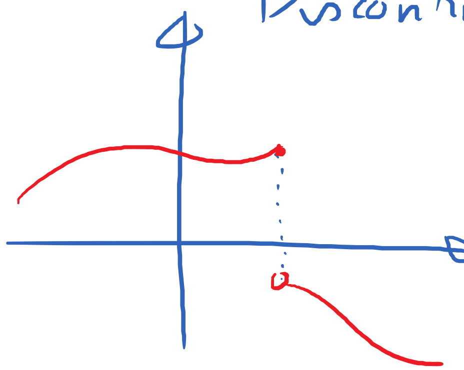
For functions given by graphs.

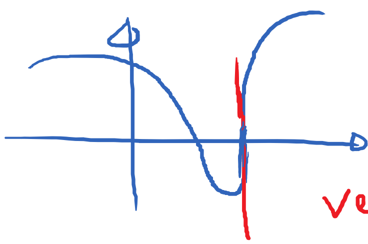
If a graph has a sharp corner at a point, then the derivative DNE there; i.e., function is not differentiable there



Corner

Discontinuity





vertical tangent line

Note: In general, differentiability is stronger than continuity. Differentiability implies continuity.