# Derivatives of Inverse Function

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Find derivative :

* polynomial functions ✓
* trig functions ✓
* Rational Functions ✓
* Composite of these functions ✓

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arcsin x ; arccos x , arctan x

**Recall:** $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$ ; $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\arccos(1) = 0$

$\arcsin(2017) = \text{DNE}$ $\quad \arctan(1) = \frac{\pi}{4}$.

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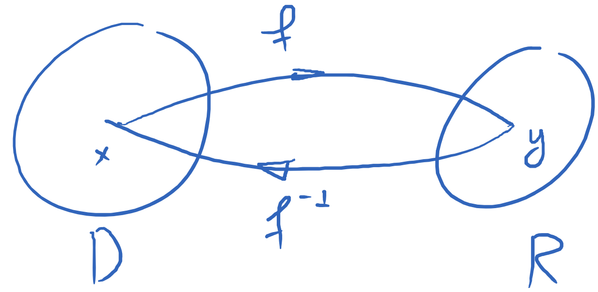
arcsin(x) gives the angle y in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin y = x$

arccos(x) gives the angle y in $[0, \pi]$ such that $\cos y = x$

arctan(x) gives the angle y in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ s.t. $\tan y = x$

In general, if $f(x) = y$ then the inverse function of $f$, denoted by $f^{-1}$ (not equal $\frac{1}{f}$) is the function that undoes what $f$ does. $f^{-1}(y) = x$



$$f^{-1}(f(x)) = x \quad ; \quad f(f^{-1}(x)) = x$$

E.g. $f(x) = x^2$ ; $f^{-1}(x) = \sqrt{x}$

$f(x) = x^3$ ; $f^{-1}(x) = \sqrt[3]{x}$

$f(x) = x + 2$ ; $f^{-1}(x) = x - 2$

$$\boxed{f(x) = x^5 + 3x^3 - 4x - 8}$$

## Inverse Function Theorem:

Let $f$ be a function that is invertible and differentiable on an interval that contains the point $x$.

Then: $$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$$

provided that $f'(f^{-1}(x)) \neq 0$

If we call $f^{-1}$ by $g$.

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

E.g. $f(x) = x^5 + 3x^3 - 4x - 8.$

$$f(1) = -8$$

$$\boxed{f^{-1}(-8) = 1}$$

Q: Find the equation of the tangent line to the graph of $f^{-1}$ at the point $(-8, 1)$

$$\text{Slope} = \frac{d}{dx}\left[f^{-1}(x)\right]\Big|_{x=-8} \underset{\text{I.F.T}}{=} \frac{1}{f'(\underbrace{f^{-1}(8)}_{1})} = \frac{1}{f'(1)} = \boxed{\frac{1}{10}}$$

$$f'(x) = 5x^4 + 9x^2 - 4$$

$$f'(1) = 10$$

E.g. $\boxed{f(-1) = 0}$ Find $(f^{-1})'(0) =$

$\boxed{f'(-1) = 1}.$

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(-1)} = \boxed{1}$$

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I.F.T: $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$.

Why is this true?

$$\boxed{f(f^{-1}(x))} = x$$

Take the derivative w.r.t. x of both sides

$$\boxed{f'(f^{-1}(x))} \cdot (f^{-1})'(x) = 1$$

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.}$$

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# Formulas for inverse trig functions

1 Find $\frac{d}{dx}\left[\arcsin x\right] = ?$

$f(x) = \arcsin x$ | $f'(x) = ?$

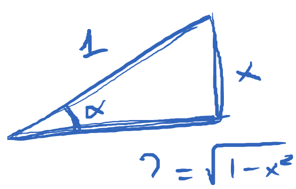
$\boxed{g(x) = \sin x} \longrightarrow g'(x) = \cos x$

$$f'(x) = \frac{1}{g'(f(x))} = \frac{1}{\cos(f(x))}$$

$$f'(x) = \frac{1}{\boxed{\cos(\arcsin x)}} = \frac{1}{\sqrt{1-x^2}}$$

$\arcsin x = \alpha$ ; $\sin \alpha$

$\downarrow$ angle whose sine $= x$




$\cos(\arcsin x)$

$= \cos(\alpha) = \sqrt{1-x^2}$

$(?)^2 + x^2 = 1$

$? = \sqrt{1-x^2}$

$$\boxed{\frac{d}{dx}\left[\arcsin x\right] = \frac{1}{\sqrt{1-x^2}}}$$