Derivatives of Inverse Trig Functions

Wednesday, July 26, 2017 7:45 AM

$$\frac{d}{dx}\left[\arcsin x\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\cos^{-1} x\right] = \boxed{\frac{-1}{\sqrt{1-x^2}}}$$

$$\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$$

look up the other formulas for $\frac{d}{dx}$ of arcsec, . . .

$$\frac{d}{dx}\left[\tan^{-1}(\text{Stuff})\right] = \frac{(\text{Stuff})'}{1+(\text{Stuff})^2}$$

E.g. $y = \tan^{-1}(x^2)$

$$\frac{dy}{dx} \stackrel{?}{=} \frac{2x}{1+x^4}$$

$y = \tan^{-1}(\sin x)$

$$\frac{dy}{dx} = \frac{\cos x}{1+\sin^2 x}$$

E.g. $y = \cos^{-1}(3x - 1)$

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-(3x-1)^2}}$$

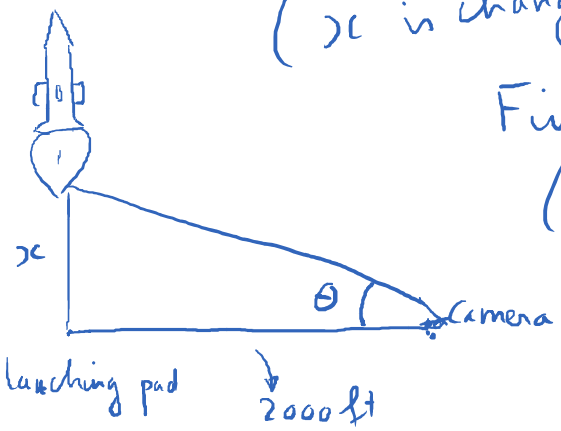
$$\frac{d}{dx}\cos^{-1}(\text{Stuff}) = \frac{-(\text{Stuff})'}{\sqrt{1-(\text{Stuff})^2}}$$

E.g. $y = x^2 \cdot \sin^{-1} x$

$$\frac{dy}{dx} = 2x \cdot \sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \cdot x^2$$

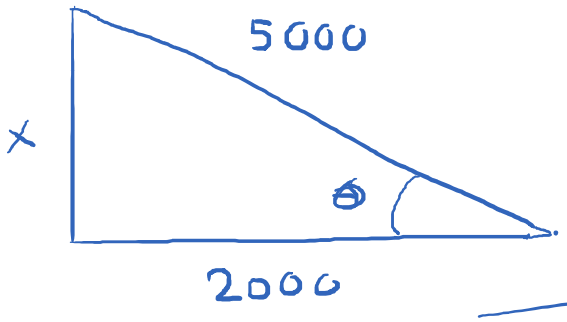
$$= \boxed{2x \cdot \sin^{-1} x + \frac{x^2}{\sqrt{1-x^2}}}$$

An application: Television camera is 2000 ft away from the launching pad of a rocket. x: height of rocket (x is changing).



Find the rate of change of θ (angle of elevation) of the camera when the rocket is 5000 ft away from the camera.

$$\tan\theta = \frac{x}{2000}.$$



$$\theta = \tan^{-1}\left(\frac{x}{2000}\right)$$

$$\text{R.O.C. of } \theta = \frac{d\theta}{dx}.$$

$$x^2 + (2000)^2 = (5000)^2$$

$$x^2 = (5000)^2 - (2000)^2$$

$$x = \sqrt{(5000)^2 - (2000)^2}$$

$$\boxed{\frac{d\theta}{dx}} = \frac{1}{1 + \left(\frac{x}{2000}\right)^2} \cdot \frac{1}{2000}$$