

3.5. Derivatives of Trig Functions

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If $f(x) = \sin x$, then $f'(x) = \cos x$.

In Leibnitz notation: $\frac{d}{dx} [\sin x] = \cos x$.

E.g. $y = x^2 \cdot \sin(x)$. Find $\frac{dy}{dx}$?

$$\frac{dy}{dx} = 2x \cdot \sin(x) + x^2 \cdot \cos(x)$$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

$$\frac{d}{dx} [\cos x] = -\sin x$$

$$\frac{d}{dx} (\tan x)$$

$$\tan x = \frac{\sin x}{\cos x} \cdot \frac{d}{dx} \left[\frac{\sin x}{\cos x} \right] = \frac{\cos x \cdot \cos x - (-\sin x) \cdot \sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\boxed{\frac{d}{dx} [\tan x] = \sec^2 x}$$

$$\frac{d}{dx} [\cot x] = -\csc^2 x.$$

E.x. $\frac{d}{dx} [\sec x] = \sec x \cdot \tan x$

$$\frac{d}{dx} [\csc x] = -\csc x \cot x$$

$$\begin{aligned} \frac{d}{dx} [\sec x] &= \frac{d}{dx} \left[\frac{1}{\cos x} \right] = \frac{0 \cdot \cos x - (-\sin x) \cdot 1}{\cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x \cdot \cos x} = \left(\frac{\sin x}{\cos x} \right) \cdot \frac{1}{\cos x} = \tan x \cdot \sec x \end{aligned}$$

$$\frac{d}{dx} [\sin x] = \cos x$$

$$\frac{d}{dx} [\cos x] = -\sin x$$

$$\frac{d}{dx} [\tan x] = \sec^2 x$$

$$\frac{d}{dx} [\cot x] = -\csc^2 x$$

$$\frac{d}{dx} [\sec x] = \sec x \cdot \tan x$$

$$\frac{d}{dx} [\csc x] = -\csc x \cdot \cot x$$

Higher order derivatives of $y = \sin x$ and $y = \cos x$

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{d^2 y}{dx^2} = -\sin x ; \frac{d^3 y}{dx^3} = -\cos x$$

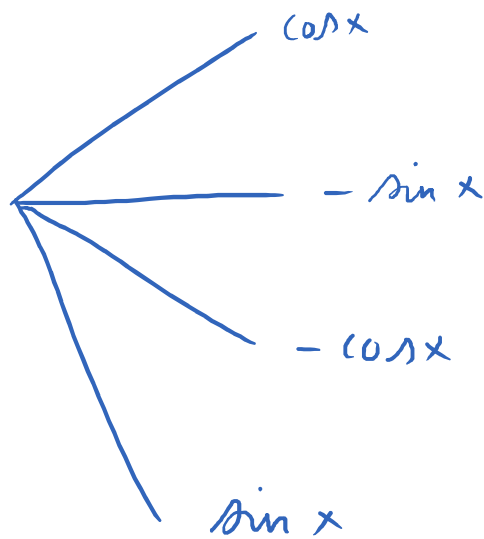
$$\frac{d^4 y}{dx^4} = \sin x ;$$

$$\frac{d^5 y}{dx^5} = \cos x$$

$$\frac{d^{2017} y}{dx^{2017}} = \cos x$$

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In general, $\frac{d^n y}{dx^n}$
($y = \sin x$)



Divide n by 4.

$$R = 1$$

$$R = 2$$

$$R = 3$$

$$R = 0$$