3.5. Derivatives of Trig Functions

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If $f(x) = \sin x$, then $f'(x) = \cos x$.

In Leibnitz notation: $\frac{d}{dx}\left[\sin x\right] = \cos x$.

E.g. $y = x^2 \cdot \sin(x)$. Find $\frac{dy}{dx}$?

$$\frac{dy}{dx} = 2x \cdot \sin(x) + x^2 \cdot \cos(x)$$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

$$\frac{d}{dx}\left[\cos x\right] = -\sin x$$

$$\frac{d}{dx}\left(\tan x\right)$$

$$\tan x = \frac{\sin x}{\cos x} \cdot \quad \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x \cdot \cos x - (-\sin x) \cdot \sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\boxed{\frac{d}{dx}\left[\tan x\right] = \sec^2 x}$$

$$\frac{d}{dx}\left[\cot x\right] = -\csc^2 x.$$

Ex. $$\frac{d}{dx}\left[\sec x\right] = \sec x \cdot \tan x$$

$$\frac{d}{dx}\left[\csc x\right] = -\csc x \cot x$$

$$\frac{d}{dx}\left[\sec x\right] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{0 \cdot \cos x - (-\sin x)\cdot 1}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x \cdot \cos x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \cdot \sec x$$

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \cdot \tan x \qquad \frac{d}{dx}[\csc x] = -\csc x \cdot \cot x.$$

Higher order derivatives of $y = \sin x$ and $y = \cos x$

$y = \sin x$

$\boxed{\frac{dy}{dx} = \cos x}$; $\frac{d^2y}{dx^2} = -\sin x$; $\frac{d^3y}{dx^3} = -\cos x$

$\frac{d^4y}{dx^4} = \sin x$; $\frac{d^5y}{dx^5} = \cos x$

$\frac{d^{2017}y}{dx^{2017}} = \cos x$

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In general, $\dfrac{d^n y}{dx^n}$ $(y = \sin x)$

Divide n by 4.

- $\cos x$ — $R = 1$
- $-\sin x$ — $R = 2$
- $-\cos x$ — $R = 3$
- $\sin x$ — $R = 0$