3.8. Implicit Differentiation

Wednesday, July 26, 2017 8:08 AM

Find Derivatives Implicitly.

So far, y = formula in x (explicit)

$y = x^2 + 2x - 3$. Find $\frac{dy}{dx}$

⟶ Explicit Differentiation

Implicit: equation: $x^2 + y^2 = 25$.

y is given implicitly in terms of x

How do we find $\frac{dy}{dx}$?

Implicit Differentiation: $x=3$; $y=4$

Given $\boxed{x^2+y^2=25}$. Find $\frac{dy}{dx}$?

Take the derivative with respect to x of both sides:

$$\frac{d}{dx}\left(x^2+y^2\right) = \frac{d}{dx}(25)$$

$$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y^2\right) = 0$$

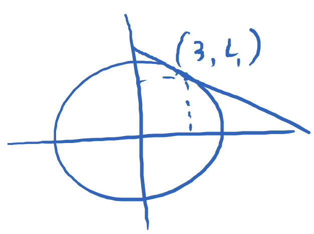
$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} \longrightarrow \boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$\frac{dy}{dx} = ?$ $x=3$; $y=4$

$\frac{dy}{dx} = -\frac{3}{4} \longrightarrow$ slope of tangent line to the curve $x^2+y^2=25$ at $(3,4)$



E.g. $4x^5 + \tan y = y^2 + 5x$.

Find $\frac{dy}{dx}$?

$$\frac{d}{dx}\left(4x^5 + \tan y\right) = \frac{d}{dx}\left(y^2 + 5x\right)$$

$$\frac{d}{dx}\left(4x^5\right) + \frac{d}{dx}\left(\tan y\right) = \frac{d}{dx}\left(y^2\right) + \frac{d}{dx}\left(5x\right)$$

$$20x^4 + \sec^2 y\,\frac{dy}{dx} = 2y\,\frac{dy}{dx} + 5$$

$$\sec^2 y\,\frac{dy}{dx} - 2y\,\frac{dy}{dx} = 5 - 20x^4$$

$$\left(\sec^2 y - 2y\right)\frac{dy}{dx} = 5 - 20x^4$$

$$\boxed{\frac{dy}{dx} = \frac{5 - 20x^4}{\sec^2 y - 2y}}$$

E.x. $x^3y + xy^3 = -8.$

Find $\dfrac{dy}{dx}$?

Solved in class.

Find the equation of the tangent line to the curve

$$x^2 + y^2 = \left(2x^2 + 2y^2 - x\right)^2$$

at the point $\left(0, \frac{1}{2}\right)$

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}\left[\left(2x^2 + 2y^2 - x\right)^2\right]$$

$$2x + 2y\frac{dy}{dx} = 2\left(2x^2 + 2y^2 - x\right) \cdot \frac{d}{dx}\left(2x^2 + 2y^2 - x\right)$$

$$\boxed{2x} + \boxed{2y\frac{dy}{dx}} = 2 \cdot \left(\boxed{2x^2} + \boxed{2y^2} - \boxed{x}\right) \cdot \left(\boxed{4x} + \boxed{4y}\frac{dy}{dx} - 1\right)$$

Plug $x = 0$; $y = \frac{1}{2}$ into this equation.

$$\frac{dy}{dx} = \underbrace{2 \cdot \left(\frac{1}{2}\right)} \cdot \left(2\frac{dy}{dx} - 1\right)$$

$$\boxed{\frac{dy}{dx}} = 2 \cdot \boxed{\frac{dy}{dx}} - 1$$

$$-\frac{dy}{dx} = -1 \; ; \quad \boxed{\frac{dy}{dx} = 1}$$

Slope $= 1$.

Point $\left(0, \frac{1}{2}\right)$.

$$y - \frac{1}{2} = x$$

$$\boxed{y = x + \frac{1}{2}}$$

Point $\left(0, \frac{1}{2}\right)$.

$$\boxed{y = x + \frac{1}{2}}$$

Find the second derivative implicitly.

$$x^2 + y^2 = 25.$$

We saw that $\frac{dy}{dx} = -\frac{x}{y}$.

Find $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(-\frac{x}{y}\right)$$

$$= -\frac{y - \boxed{\frac{dy}{dx}} \cdot x}{y^2}$$

$$= -\frac{y - \left(-\frac{x}{y}\right)\cdot x}{y^2}$$

$$= -\frac{\boxed{y + \frac{x^2}{y}}}{y^2} = -\frac{\boxed{\frac{y^2+x^2}{y}}}{\boxed{y^2}}$$

$$\frac{d^2y}{dx^2} = -\frac{y^2+x^2}{y}\cdot\frac{1}{y^2} = \boxed{-\frac{y^2+x^2}{y^3}}$$