Integration By Substitution (u-sub)

Thursday, August 10, 2017 10:33 AM

$$\int \sin(x)\,dx = -\cos x + C \qquad \int \sin(u)\,du = -\cos(u) + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

$(n \neq -1)$

Substitution Method (u-sub)

It works if the integral has the form

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$$

$g(x)$ → u ; $g'(x)\,dx$ → du

let $u = g(x)$. $\quad du = g'(x)\,dx$

E.g. $\int [\sin(x^3)] \cdot (3x^2)\,dx$

f , u , du

let $u = x^3$. $du = 3x^2\,dx$

$$\int \sin(u)\,du = -\cos(u) + C$$

$$\int \sin(x^3)\cdot 3x^2\,dx = -\cos(x^3) + C$$

Check: $\frac{d}{dx}[-\cos(x^3)] = \sin(x^3)\cdot 3x^2$

E.g. $\int \sin(x^3)\cdot 9x^2\,dx = 3\cdot\int \sin(x^3)\cdot 3x^2\,dx$

u , du

$u = x^3$. $du = 3x^2\,dx$

$$3\cdot\int \sin(u)\,du = -3\cos(u) + C$$

$$= -3\cos(x^3) + C$$

E.g. $\frac{1}{3}\int \sin(x^3)\,3x^2\,dx$

$x^3 \to u$, $3x^2\,dx \to du$

$= \frac{1}{3}\int \sin(u)\,du = -\frac{1}{3}\cos(u) + C$

$= -\frac{1}{3}\cos(x^3) + C.$

E.g. $\frac{1}{3}\int \frac{3x^2\,dx}{x^3+6}$ ($3x^2\,dx \to du$, $x^3+6 \to u$)

let $u = x^3 + 6$. $du = 3x^2\,dx$

$\frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|x^3+6| + C.$

Check: $\frac{1}{3}\cdot\frac{3x^2}{x^3+6} = \frac{x^2}{x^3+6}$

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$$\int \tan(x)\,dx = -\int \frac{-\sin(x)\,dx}{\cos(x)}$$

du over $-\sin(x)\,dx$; u over $\cos(x)$

$u = \cos(x)$; $du = -\sin x\,dx$

$$-\int \frac{du}{u} = -\ln|u| + C = -\ln|\cos x| + C$$

$$= \ln\left|(\cos x)^{-1}\right| + C$$

$$= \ln|\sec x| + C$$

$$\int \cot^2 x\,dx = \int (\csc^2 x - 1)\,dx = -\cot x - x + C$$

$$\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x \longrightarrow \csc^2 x - 1 .$$

$$1 + \tan^2 x = \sec^2 x$$

E.x.

1) $\int \cos^2(x) \cdot \sin(x)\,dx$

2) $\int z \cdot \sqrt{z^2 - 5}\,dz$

3) $\int \frac{\sin(t)}{\cos^3(t)}\,dt$

Solved in class.