2.3. Limit Laws

Thursday, July 13, 2017 10:48 AM

Goals: 1 Use limit Laws to find limits

2 Limits of the form $\frac{0}{0}$.

3 Squeeze Theorem

Recall: $\lim_{x \to 1} \boxed{\frac{x^2 - 1}{x - 1}}$

x	$y = \frac{x^2-1}{x-1}$
0.9	...
0.99	...
0.999	...

$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1}$

x	$y = \frac{x^2-1}{x-1}$
1.1	...
1.01	...
1.001	...

$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1}$

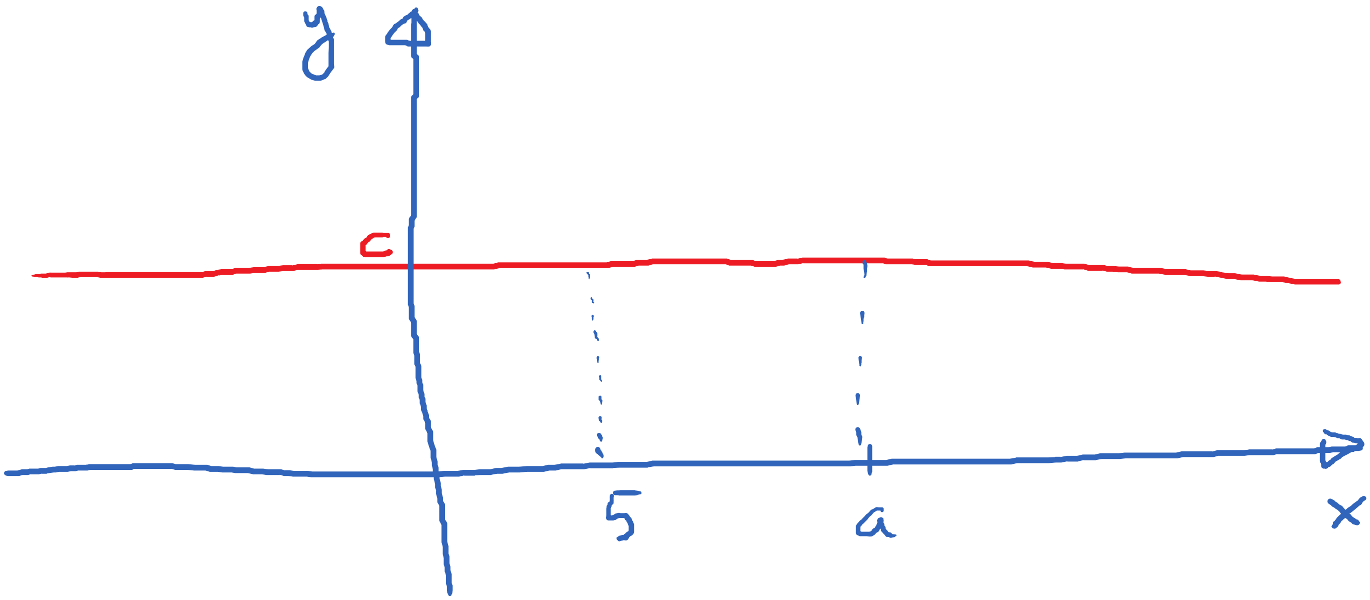
Monday, July 17, 2017 7:39 AM

Find the limit numerically.

$\longrightarrow$ Today, find limits analytically.

2 basic limits:

1 c : constant. $f(x) = c$.

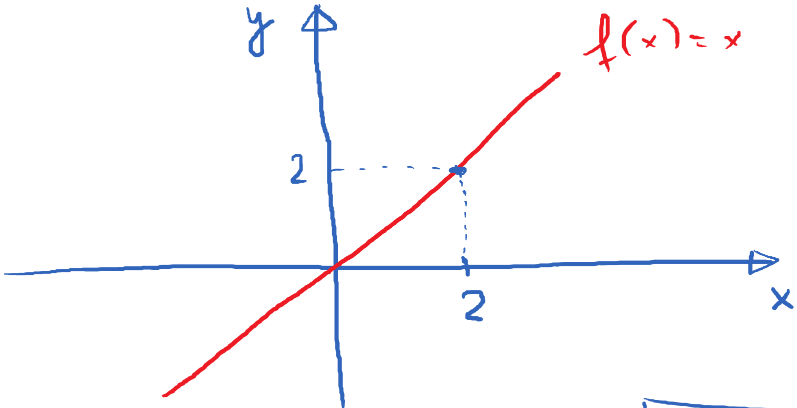


$$\lim_{x \to 5} f(x) = c.$$

$$\lim_{x \to a} f(x) = c.$$

$$\boxed{\lim_{x \to a} c = c}$$

(2) $f(x) = x$ (Identity function)



$$\lim_{x \to 2} f(x) = 2$$

$$\lim_{y \to -10000} f(x) = -10000$$

$$\lim_{x \to a} f(x) = a.$$

$$\boxed{\lim_{x \to a} x = a}$$

$$\boxed{\lim_{x \to a} c = c \quad ; \quad \lim_{x \to a} x = a}$$

Limit Laws:

1. Sum law (and Difference): $f(x), g(x)$: function

Given: $\lim_{x \to a} f(x) = L$; $\lim_{x \to a} g(x) = M$ (L, M: finite #)

$$\lim_{x \to a} \left(f(x) \pm g(x) \right) = L \pm M$$

② Product Law

$$\lim_{x \to a} [f(x) \cdot g(x)] = \left(\lim_{x \to a} f(x)\right) \cdot \left(\lim_{x \to a} g(x)\right)$$

$$= L \cdot M$$

E.g. $\lim_{x \to 5} x = 5$

$\lim_{x \to 5} x^2 = 25$ (Product law)

$\lim_{x \to 5} x^3 = 125$

$\lim_{x \to 5} x^n = 5^n$

$\lim_{x \to 5} (x + x^3) = 130$ (Sum law)

$$\lim_{x \to 5} (x^2 - x^3) = 25 - 125 = -100$$

③ Constant Multiple law.

$$\lim_{x \to a} f(x) = L. \qquad c : \text{constant}.$$

$$\lim_{x \to a} (c \cdot f(x)) = c \cdot \lim_{x \to a} f(x) = c \cdot L$$

E.g. $\lim_{x \to 5} x^2 = 25.$ $\quad \lim_{x \to 5} (10x^2) = 250$

$$\lim_{x \to 5} (x^2 - x^3) = -100$$

$$\lim_{x \to 5} \left(-\frac{1}{20}(x^2 - x^3)\right) = -\frac{1}{20} \cdot (-100) = 5.$$

(4) Quotient Law:

$$\lim_{x\to a} f(x) = L \ ; \ \lim_{x\to a} g(x) = M.$$

$(L, M:$ finite # ; $M \neq 0)$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{L}{M}$$

(5) Root law: $\lim_{x\to a} f(x) = L$.

Then $\lim_{x\to a} \sqrt{f(x)} = \sqrt{L}$; $\lim_{x\to a} \sqrt[3]{f(x)} = \sqrt[3]{L}$.

$\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{L}$ (provided $\sqrt[n]{L}$ makes sense.)

⑥ Power Law: $\lim_{x \to a} [f(x)]^n = L^n$

E.g. $f(x) = 10x^7 - x^5 + \frac{2}{3}x^2$

$$\lim_{x \to 1} f(x) = 10 - 1 + \frac{2}{3} = \cdots$$

$g(x) = \sqrt{15}$ $\quad \lim_{x \to 2017} g(x) = \sqrt{15}$