$$h(x) = \sqrt{x^2 + 2x - 15}$$

$$\lim_{x \to 5} h(x) = \sqrt{25 + 10 - 15} = \sqrt{20}$$

Bottom line: When you try to find the limit $\lim_{x \to a} f(x)$; you plug the # a into the formula $f(x)$, if you get out a finite #; that is the answer.

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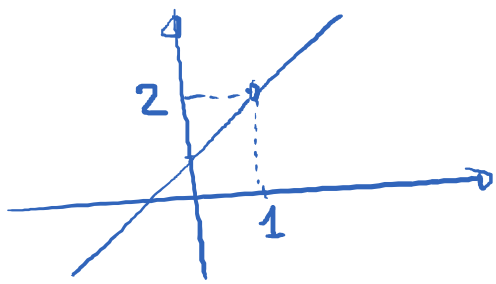
What if we don't get a finite #?

E.g. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0} \to$ limit of the form $\frac{0}{0}$.

$$\frac{x^2 - 1}{x - 1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}} = x + 1$$



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A strategy to find $\frac{0}{0}$ limits: ($\lim_{x \to a} \frac{f(x)}{g(x)}$ where $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$)

1. Factor the numerator and denominator completely
2. Cancel the common factors
3. Plug $x = a$ into the simplified function.

E.g. 1 $\lim\limits_{x \to -3} \dfrac{x^2 + 4x + 3}{x^2 - 9} = \dfrac{0}{0}$

Find the limit analytically.

$$= \lim_{x \to -3} \frac{(x+1)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)} = \lim_{x \to -3} \frac{x+1}{x-3} = \frac{-2}{-6} = \boxed{\frac{1}{3}}$$

E.x. $\lim\limits_{h \to 0} \dfrac{(1+h)^2 - 1}{h} \left(= \dfrac{0}{0}. \right)$

$$= \lim_{h \to 0} \frac{(1+h)(1+h) - 1}{h} = \lim_{h \to 0} \frac{h^2 + 2h + 1 - 1}{h}$$

$$= \lim_{h \to 0} \frac{h^2 + 2h}{h} = \lim_{h \to 0} \frac{h(h+2)}{h} = \lim_{h \to 0} (h+2) = \boxed{2}.$$

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Some variants of this technique:

$\frac{0}{0}$ limits that involve radicals.

E.g. $\lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x - 5} \quad \left(= \frac{0}{0} \right)$

Multiply both numerator and denominator by the conjugate of $\sqrt{x-1} - 2$, which is $\sqrt{x-1} + 2$.

$= \lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x - 5} \cdot \frac{\sqrt{x-1} + 2}{\sqrt{x-1} + 2} = \lim_{x \to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x-1} + 2)}$

$= \lim_{x \to 5} \frac{1}{\sqrt{x-1} + 2} = \boxed{\frac{1}{4}}$

* limits that involve complex fraction

E.g. $$\lim_{h \to 0} \frac{\frac{1 \cdot 5}{(5+h) \cdot 5} - \frac{1(5+h)}{5 \cdot (5+h)}}{h} \quad \left(= \frac{0}{0} \right)$$

$$= \lim_{h \to 0} \frac{\frac{5}{(5+h)5} - \frac{5+h}{5(5+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{5 - (5+h)}{5(5+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-h}{5(5+h)}}{h} = \lim_{h \to 0} \frac{-h}{5(5+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-1}{5(5+h)} = \boxed{-\frac{1}{25}}$$

One-sided limits

$$f(x) = \begin{cases} -x-2 & \text{if } x < -1 \\ 2 & \text{if } x = -1 \\ x^3 & \text{if } x > -1 \end{cases}$$

if $x > -1$

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} (-x-2) = \boxed{0}$$

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1} (-x-2) = -1$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1} (x^3) = -1$$

$$\lim_{x \to -1} f(x) = -1. \qquad f(-1) = 2$$

* Limits of the form $\frac{K}{0}$ where $K \neq 0$

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$$\lim_{x \to 10^+} \frac{2}{x-10} = \infty$$

$$\lim_{x \to 10^-} \frac{2}{x-10} = -\infty$$

$$\lim_{x \to 2^-} \frac{-100}{2-x} = -\infty$$

Squeeze Theorem.

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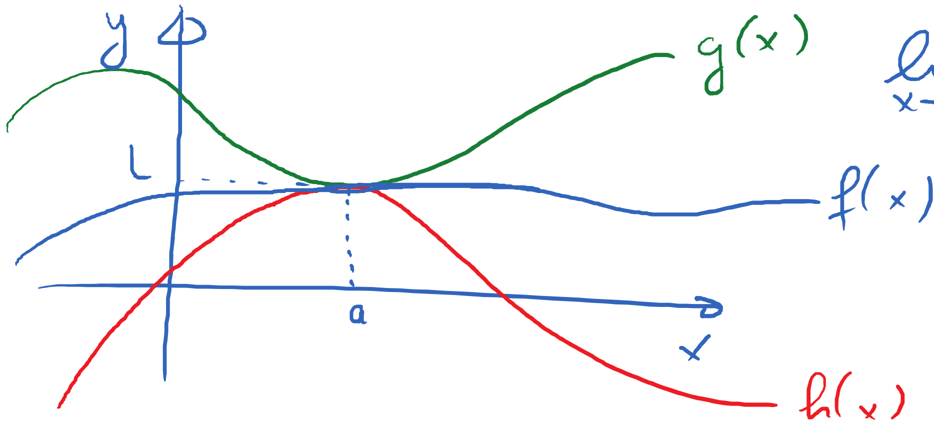
$$\lim_{x \to 0} x^2 \cdot \sin\left(\frac{1}{x}\right) \longleftarrow \text{Squeeze Theorem}$$

$$h(x) \leq f(x) \leq g(x)$$

$$\lim_{x \to a} f(x) = ?$$



If $\lim_{x \to a} h(x) = \lim_{x \to a} g(x) = L$

then $\lim_{x \to a} f(x) = L$.

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$$\lim_{x \to 0} x^2 \cdot \sin\left(\frac{1}{x}\right) = ?$$ by Squeeze Theorem

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$-x^2 \leq x^2 \cdot \sin\left(\frac{1}{x}\right) \leq x^2$$

$\lim_{x \to 0}$ 0

Hence, by Squeeze Theorem, $\lim_{x \to 0} x^2 \cdot \sin\left(\frac{1}{x}\right) = 0$