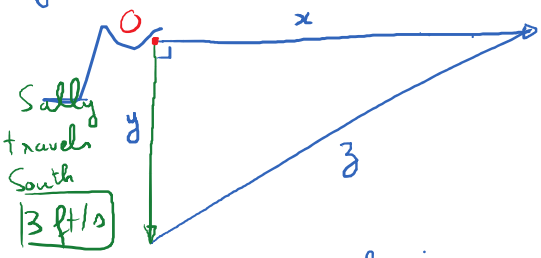
4.1. Related Rates Problems

Thursday, July 27, 2017 11:00 AM

E.g. Bob travels east. 4 ft/s



10 s later

Q: At what rate is the distance between Bob and Sally changing

x: distance from Bob to O.

y: distance from Sally to O.

Both x, y are functions of time

$\frac{dx}{dt} = 4 \; ; \; \frac{dy}{dt} = 3$

z: distance between Bob and Sally

z is a function of time.

3 quantities that are changing with time x, y, z.

Know: R.O.C. of x, y

Want: R.O.C. of z

$\frac{dz}{dt} = ?$ when $t = 10s$

$x^2 + y^2 = z^2$

Relationships among the quantities

Take derivative w.r.t time t of both sides:

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(z^2)$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

($x = 40$, $\frac{dx}{dt} = 4$; $y = 30$, $\frac{dy}{dt} = 3$; $z = 50$, $\frac{dz}{dt} = ?$)

$$2 \cdot 40 \cdot 4 + 2 \cdot 30 \cdot 3 = 2 \cdot 50 \cdot \frac{dz}{dt}$$

$$320 + 180 = 100\frac{dz}{dt}$$

$$500 = 100\frac{dz}{dt}$$

$$\frac{dz}{dt} = 5 \text{ ft/s}$$

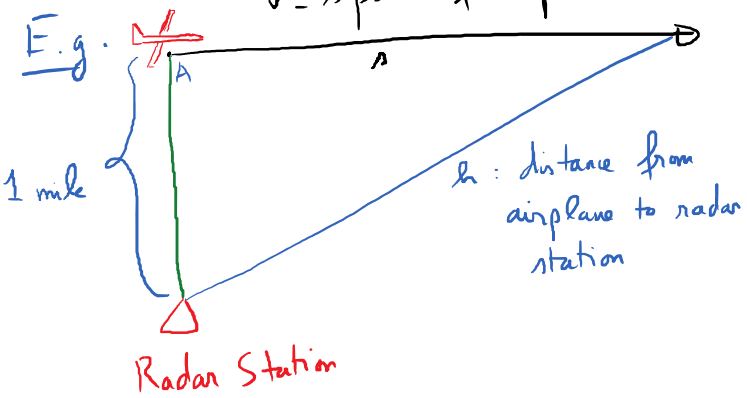
At $t = 10s$, $x = 40$ ft, $y = 30$ ft

$z = ?$ $\quad x^2 + y^2 = z^2$; $\quad 1600 + 900 = z^2$

$2500 = z^2$

$z = 50$

E.g. v = speed of airplane = 500 mi/h



s : distance from airplane to A.

$\frac{ds}{dt} = 500$

Find the rate at which the distance from the plane to the station is changing when the plane is 2 mi away from the station.

($\frac{dh}{dt} = ?$ when $h = 2$ mi)

Relation between s and h

$$1 + s^2 = h^2$$

$$\frac{d}{dt}(1 + s^2) = \frac{d}{dt}(h^2)$$

$$2s \cdot \frac{ds}{dt} = 2h \frac{dh}{dt}$$

when: $h = 2$; $1 + s^2 = 4$

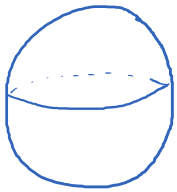
$s^2 = 3$

$s = \sqrt{3}$

$$2 \cdot \sqrt{3} \cdot 500 = 2 \cdot 2 \cdot \frac{dh}{dt}$$

$$1000\sqrt{3} = 4\frac{dh}{dt} \; ; \quad \frac{dh}{dt} = \frac{1000\sqrt{3}}{4} = 250\sqrt{3}.$$

E.g.



Snowball is melting.

Spherical Snowball.

Volume of Snowball is decreasing at a rate of $2 \frac{cm^3}{min}$

Q: How fast is the radius of the snow ball is changing when the radius is 5 cm.

Volume, Radius: quantities that are changing with time

V Q

$$V = \frac{4}{3}\pi \cdot Q^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3Q^2 \frac{dQ}{dt}$$

$$-2 = \frac{4}{3}\pi \cdot 3 \cdot (5)^2 \cdot \frac{dQ}{dt}$$

$$-2 = \frac{4}{3}\pi \cdot 3 \cdot 25 \cdot \frac{dQ}{dt}$$

$$-2 = 100\pi \frac{dQ}{dt}.$$

$$\frac{dQ}{dt} = \frac{-2}{100\pi} = \frac{-1}{50\pi}$$

Rate of change of radius: $\frac{1}{50\pi}$

$$-2 = \frac{dQ}{dt}$$

$$\frac{dQ}{dt} = \frac{-2}{100\pi} = \frac{-1}{50\pi}$$

Rate of change of radius : $\boxed{\frac{1}{50\pi}}$

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