Related Rates Problems (cont.)

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(1) 2 or more quantities. These quantities are changing with respect to time.

(2) These quantities are related by an equation or set of equations.

(3) Know (given) the rate of change of all but one quantity.

(4) Find the rate of change of the remaining quantity.

How approach these problems

(1) Identify all changing quantities

(2) Name them using variables such as $x, y, z, \ldots$

→ keep in mind these are functions of time

(3) Find the relationship (equations) that relate these quantities

(4) Differentiate w.r.t. time t.

(5) Use the given information to find the missing R.O.C.

E.g. Parallel

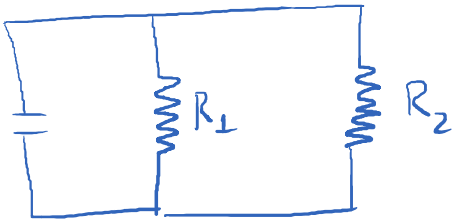
2 resistors with resistances R_1 and R_2.

R: total resistance.

R_1: increasing at a rate of 0.3 Ω/s

R_2: ——————— 0.2 Ω/s.

How fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?



(1) Changing quantities: R, R_1, R_2

(2) → Name

(3) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ ← relation

$\frac{dR_1}{dt} = 0.3\ \Omega/s$, $\frac{dR_2}{dt} = 0.2\ \Omega/s$, $\frac{dR}{dt} = ?$

(4) Differentiate the relation with respect to time:

$$\frac{d}{dt}\left(\frac{1}{R}\right) = \frac{d}{dt}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$-\frac{1}{R^2}\frac{dR}{dt} = -\frac{1}{R_1^2}\cdot\frac{dR_1}{dt} - \frac{1}{R_2^2}\cdot\frac{dR_2}{dt}$$

$$\frac{1}{R^2}\cdot\frac{dR}{dt} = \frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}$$

$\frac{dR}{dt}$: ? $\frac{dR_1}{dt}$: 0.3 $\frac{dR_2}{dt}$: 0.2

$R_1 = 80$ $R_2 = 100$

(5) Need R.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \frac{1}{R} = \frac{1}{80} + \frac{1}{100} = \frac{10+8}{800} = \frac{18}{800} = \frac{9}{400}$$

$$\rightarrow R = \frac{400}{9}$$

$$\frac{1}{\left(\frac{400}{9}\right)^2}\cdot\frac{dR}{dt} = \frac{1}{(80)^2}\cdot(0.3) + \frac{1}{(100)^2}\cdot(0.2)$$

$\frac{dR}{dt}$: ?

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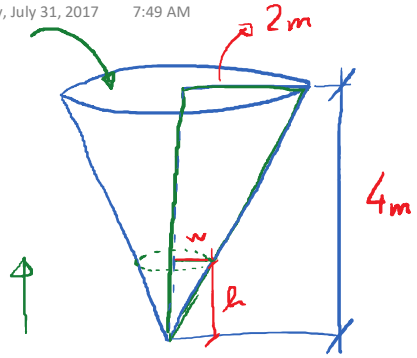
E.g.



Water tank. Inverted Circular Cone.

Base radius: 2m

Height: 4m.

Water is being pumped into the tank at a rate of

$2\ m^3/min \longrightarrow \frac{dV}{dt}$

Q: Find the rate at which the water level is rising when the water reaches a depth of 3m.

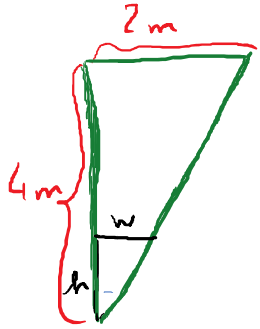
$\frac{dh}{dt}$

(1) Volume, height of water, radius of water surface.

(2) V, h, w

(3) Relation: $V = \pi w^2 \cdot \frac{h}{3} = \frac{1}{3}\pi w^2 \cdot h$

$V = \frac{1}{3}\pi w^2 \cdot h$



Similar triangles

$\frac{h}{4} = \frac{w}{2} \longrightarrow w = \frac{h}{2}$

$V = \frac{1}{3}\pi \cdot \left(\frac{h}{2}\right)^2 \cdot h = \frac{1}{3}\pi \cdot \frac{h^2}{4} \cdot h$

$V = \frac{1}{3}\pi \cdot \frac{h^3}{4} = \frac{1}{12}\pi h^3$

$V = \frac{1}{12}\pi \cdot h^3$

(4) $\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$?

$2 = \frac{9\pi}{4} \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{9\pi}$ m/min

$2 \qquad \dot{3}$

$2 = \frac{\pi}{12} \cdot 3 \cdot (3)^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{9\pi}$ m/min

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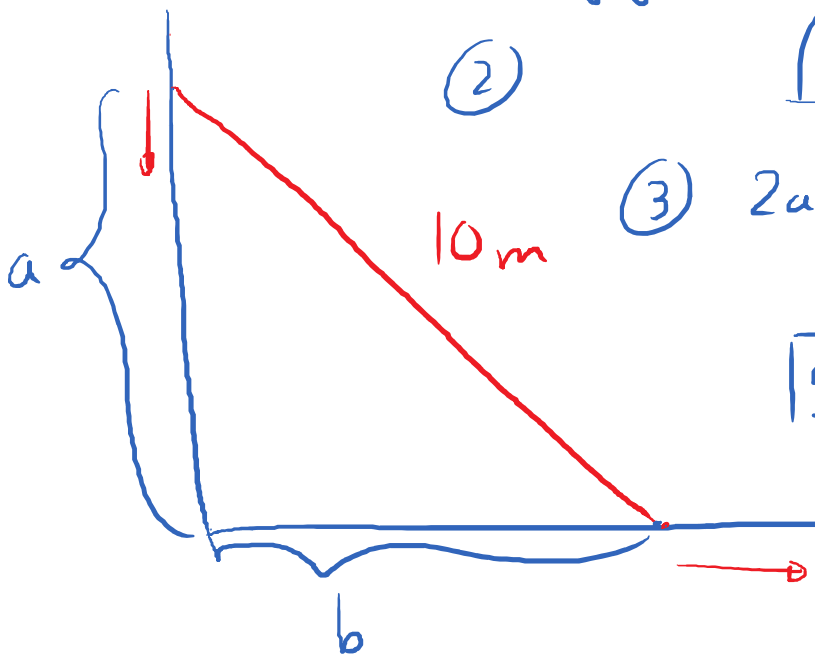

① Changing Quantities : b, a

②
$$a^2 + b^2 = 100$$

③
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$\boxed{a}\boxed{\frac{da}{dt}} + \underset{6}{\boxed{b}}\boxed{\frac{db}{dt}} = 0$$

$$\frac{db}{dt} = 0.4$$

when $b = 6$

$$a^2 + 36 = 100$$

$$a^2 = 64 \; ; \; a = 8$$

$$8 \cdot \frac{da}{dt} + 6 \cdot (0.4) = 0$$

$$8\frac{da}{dt} = -2.4$$

$$\frac{da}{dt} = -0.3$$