4.4. Rolle's Theorem and Mean Value Theorem.

Tuesday, August 1, 2017 10:32 AM

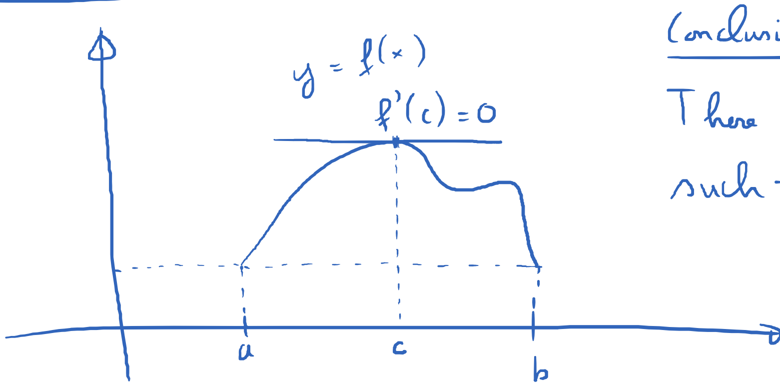
Existence Theorem.

Rolle's Theorem:

f : function on the interval $[a,b]$

1. f is continuous on $[a,b]$
2. f is differentiable on (a,b)
3. $f(a) = f(b)$

Hypothesis of Theorem



Conclusion:

There exists a number c in (a,b) such that $f'(c) = 0$

E.g. $f(x) = x^3 - 4x$ over $[-2, 2]$.

ⓐ Verify that f satisfies all conditions (requirements) of Rolle's Theorem

① Continuous on $[-2, 2]$

② Differentiable on $(-2, 2)$

③ $f(2) = 8 - 8 = 0$; $f(-2) = -8 + 8 = 0$

$f(2) = f(-2)$ ✓

⟶ Rolle's Theorem applies.

⟶ there exists c s.t. $f'(c) = 0$

ⓑ Find the value(s) of c guaranteed by Rolle's Theorem.

$$f'(x) = 3x^2 - 4$$

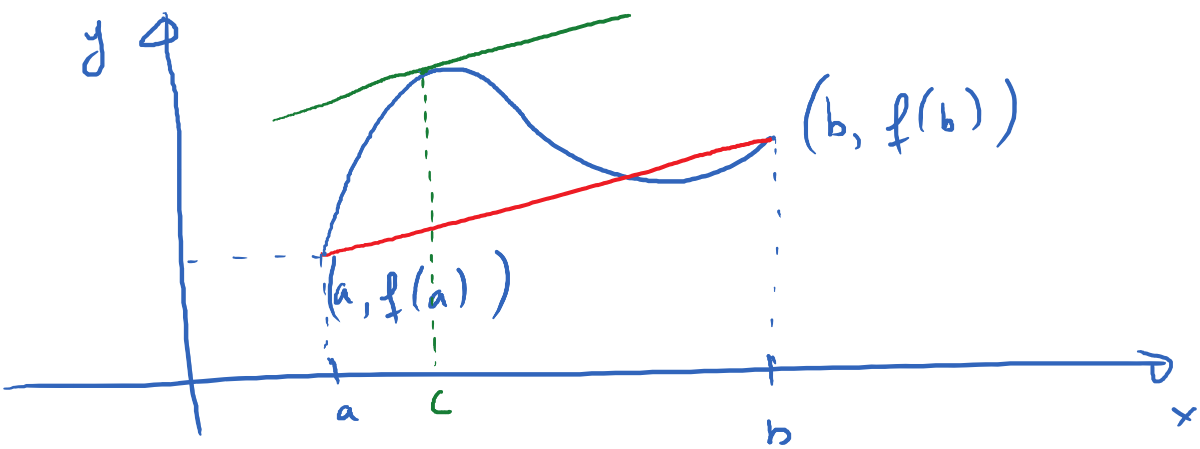
$$f'(c) = 3c^2 - 4 = 0$$

$$c^2 = \frac{4}{3} \; ; \quad c = \pm\sqrt{\frac{4}{3}} = \pm \frac{2}{\sqrt{3}}$$

Mean Value Theorem (MVT)

f: function on $[a, b]$
f: differentiable on (a, b)
f: continuous on $[a, b]$

Hypothesis



Conclusion: there exists a point c in (a,b) s.t

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

E.g. T or F

At 2pm: speedometer reads 30 mi/h.

At 2:10pm: speedometer reads: 50 mi/h

T or F: At some point in (2pm, 2:10pm), the acceleration of car is exactly 120 mi/h²

$v(2pm) = 30$; $v(2{:}10pm) = 50$

MVT there exists a time c in (2, 2:10) s.t.

$$v'(c) = \frac{v(2{:}10) - v(2)}{\frac{1}{6}}$$

$$\underbrace{v'(c)}_{a(c)} = \frac{50 - 30}{\frac{1}{6}} = \frac{20}{\frac{1}{6}} = 120$$

E.g. $f(x) = \ln x$. on $[1, 4]$

1. Verify all conditions of MVT are satisfied.
2. Find the value(s) of c guaranteed by the MVT

Solved in class.