The Chain Rule

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Goal: Understand the chain rule and use it to find derivatives.

$$\boxed{\frac{d}{dx}\left[x^2\right] = 2x}$$

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$$\frac{d}{dx}\left[\sqrt{x}\right] = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\left[\frac{1}{x}\right] = -\frac{1}{x^2}$$

$$\frac{d}{dx}\left[\sin x\right] = \cos x$$

$$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x$$

$$\frac{d}{dx}\left[\left(x^3 + \sin x\right)^2\right]$$

$$\frac{d}{dx}\left[\left(\sqrt{x} + \frac{1}{x} + \cos x\right)^n\right]$$

$$\frac{d}{dx}\left(\sqrt{3x^2 + 10x - 5}\right)$$

$$dx\left[\frac{1}{\cos x + \sin x}\right]$$

$$\frac{d}{dx}\left[\sin\left(x^3 + 2x^2 - 10\right)\right]$$

$$\frac{d}{dx}\left[\sec\left(2\sqrt{x} - \frac{3}{x}\right)\right]$$

$$\frac{d}{dx}\left[\left(\boxed{\text{Stuff}}\right)^{\boxed{2}}\right] = \underbrace{2\cdot(\text{Stuff})}_{\text{R.O.C w.r.t. Stuff}} \cdot \overbrace{\frac{d}{dx}\left(\boxed{\text{Stuff}}\right)}^{\text{R.O.C w.r.t. } x}$$

$$\frac{d}{dx}\left[\left(\text{Stuff}\right)^n\right]$$

$$\frac{d}{dx}\left(\sqrt{\text{Stuff}}\right)$$

$$\frac{d}{dx}\left[\frac{1}{\text{Stuff}}\right]$$

$$\frac{d}{dx}\left(\sin(\text{Stuff})\right)$$

$$\frac{d}{dx}\left[\sec(\text{Stuff})\right] = \sec(\text{Stuff}) \cdot \tan(\text{Stuff}) \cdot \frac{d}{dx}(\text{Stuff})$$

E.g.

$$\frac{d}{dx}\left[\left(\boxed{x^3 + \sin x}\right)^{\boxed{2}}\right] = \underbrace{2\cdot\left(x^3 + \sin x\right)}_{\substack{\text{derivative of outside}\\ \text{w.r.t. inside}}} \cdot \underbrace{\left(3x^2 + \cos x\right)}_{\substack{\text{deri. of inside}\\ \text{w.r.t. } x}}$$

$$\frac{d}{dx}\left[\left(\boxed{x^3 - 2x^2 + 5}\right)^{\boxed{10}}\right] = 10\cdot\left(x^3 - 2x^2 + 5\right)^9 \cdot \left(3x^2 - 4x\right)$$

$$\frac{d}{dx}\left[\sqrt{\underbrace{3x^2 + 10x - 5}_{\text{Stuff}}}\right] = \frac{1}{2\sqrt{3x^2 + 10x - 5}} \cdot (6x + 10)$$

$$\frac{d}{dx}\left[\sec\left(\underbrace{\boxed{2\sqrt{x} - \frac{3}{x}}}_{\text{Stuff}}\right)\right] = \sec\left(2\sqrt{x} - \frac{3}{x}\right) \cdot \tan\left(\boxed{2\sqrt{x} - \frac{3}{x}}\right) \cdot \left(\frac{1}{\sqrt{x}} + \frac{3}{x^2}\right)$$

$$\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}} \qquad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

Ex.

(1) $$\frac{d}{dx}\left[\frac{1}{\cos x + x^2 + 7}\right]$$

(2) $$\frac{d}{dx}\left[\overset{y}{\boxed{\tan}}\left(\overset{u}{x + \frac{1}{x}}\right)\right]$$

(1) $$\frac{d}{dx}\left(\frac{1}{\text{Stuff}}\right) = -\frac{1}{(\text{Stuff})^2} \cdot \frac{d}{dx}(\text{Stuff})$$

$$-\frac{1}{\left(\cos x + x^2 + 7\right)^2} \cdot \left(-\sin x + 2x\right)$$

$$= -\frac{\left(-\sin x + 2x\right)}{\left(\cos x + x^2 + 7\right)^2} = \frac{\sin x - 2x}{\left(\cos x + x^2 + 7\right)^2}$$

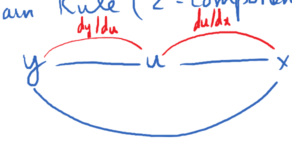
(2) $$\frac{d}{dx}\left(\tan\left(\boxed{\text{Stuff}}\right)\right) = \sec^2(\text{Stuff}) \cdot \frac{d}{dx}(\text{Stuff})$$

$$\left(\sec^2\left(x + \frac{1}{x}\right)\right) \cdot \left(1 - \frac{1}{x^2}\right)$$

(2) $\frac{d}{dx}\left(\tan(\underline{\text{Stuff}})\right) = \sec^2(\text{Stuff}) \cdot \frac{d}{dx}(\text{Stuff})$

$$\left(\sec^2\left(x+\frac{1}{x}\right)\right)\cdot\left(1-\frac{1}{x^2}\right)$$

Chain Rule (2-component)



$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\left[\underset{y}{\tan}\left(\underset{u}{x+\frac{1}{x}}\right)\right]$$

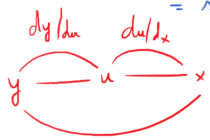
$y = \tan(u) \longrightarrow \frac{dy}{du} = \boxed{\sec^2(u)}$

$u = x + \frac{1}{x} \longrightarrow \frac{du}{dx} = 1 - \frac{1}{x^2}$

$$\frac{dy}{dx} = \sec^2(u)\cdot\left(1-\frac{1}{x^2}\right)$$

$$= \sec^2\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$$

E.g. $\frac{d}{dx}\left(\sqrt{\underset{u}{\sin x + \cos x}}\right)$



$y = \sqrt{u}$

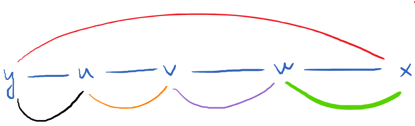
$\boxed{u = \sin x + \cos x}$

$\frac{dy}{du} = \frac{1}{2\sqrt{u}}$ $\quad \frac{du}{dx} = \cos x - \sin x$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$= \frac{1}{2\sqrt{u}}\cdot(\cos x - \sin x)$$

$$= \frac{1}{2\sqrt{\sin x + \cos x}}\cdot(\cos x - \sin x)$$



$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dw}\cdot\frac{dw}{dx}$$

E.g. $y = \sin(\underset{u}{\cos}(\sqrt{x}))$. $\frac{dy}{dx} = ?$

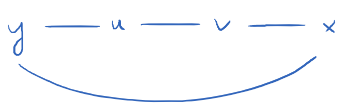
$v = \sqrt{x}$

$u = \cos(v)$

$y = \sin(u)$

$\frac{dv}{dx} = \frac{1}{2\sqrt{x}}$; $\frac{du}{dv} = -\sin(v)$

$\frac{dy}{du} = \cos(u)$



$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx}$$

$$= \cos(u)\cdot(-\sin(v))\cdot\frac{1}{2\sqrt{x}}$$

$$= \boxed{\cos(\cos(\sqrt{x}))\left(-\sin(\sqrt{x})\right)\cdot\frac{1}{2\sqrt{x}}}$$

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E.g. 1 $y = \sin(\sin(\sin(\sin(x))))$. $\frac{dy}{dx} = ?$

2 $y = \cos\left(\sqrt{\sin(\tan(\pi x))}\right)$ $\frac{dy}{dx} = ?$

* 3 $y = \left[x + (x + \sin^2 x)^3\right]^4$ $\frac{dy}{dx} = ?$