1.5. Quadratic Equations

Thursday, September 7, 2017 10:01 AM

A quadratic equation is an equation of the form

$$ax^2 + bx + c = 0$$

where a, b, c are constant.

$a \neq 0$.

Objective 1: Solve quadratic equations by factoring.

E.g. $3x^2 - 9x = 0$

$$3x(x-3) = 0$$

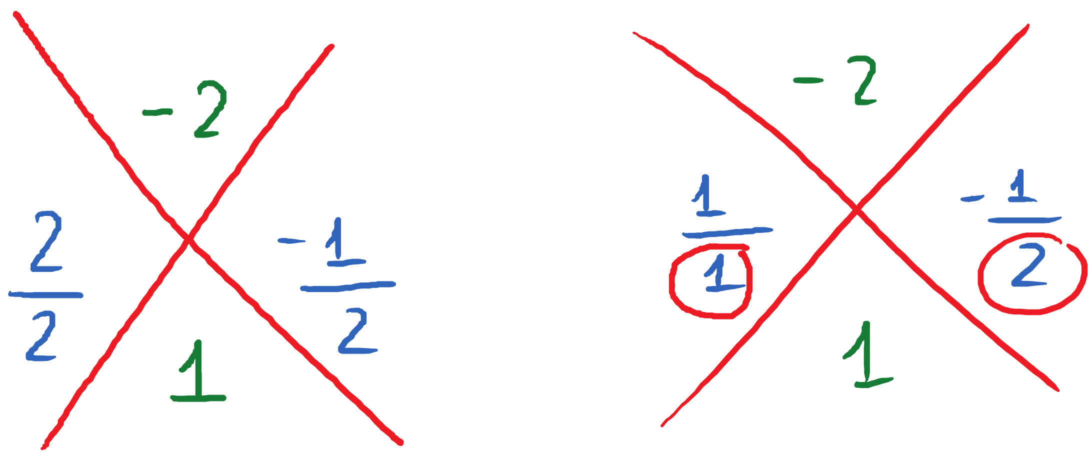
$3x = 0$ or $x - 3 = 0$

$\boxed{x = 0}$ $\boxed{x = 3}$

E.g.

$$2x^2 + x = 1$$

$$2x^2 + x - 1 = 0$$



$$(1x + 1)(2x - 1) = 0$$

$$(x + 1)(2x - 1) = 0$$

$x + 1 = 0$ or $2x - 1 = 0$

$\boxed{x = -1}$ $\boxed{x = \frac{1}{2}}$

Objective 2: Solve quadratic equations by the square root property.

E.g. $3x^2 - 21 = 0$

$$3x^2 = 21$$

$$x^2 = 7$$

$$x = \boxed{\pm\sqrt{7}}$$

E.g. $5x^2 + 45 = 0$

$$5x^2 = -45$$

$$x^2 = -9$$

$$x = \boxed{\pm 3i}$$

E.g. $3\cdot(x-4)^2 = 15$

$$(x-4)^2 = 5$$

$$x - 4 = \pm\sqrt{5}$$

$$x = 4 \pm \sqrt{5}$$

Solution set: $\{4+\sqrt{5},\ 4-\sqrt{5}\}$.

Objective 3: Solving quadratic equations by completing the square

Review of how to complete the square

Expression: $x^2 + 6x$.

Q: What should be added to this expression

——— so that it becomes a perfect square.

$$x^2 + 6x + 9$$

$$= (x+3)(x+3) = (x+3)^2$$

$$x^2 + 10x + 25$$

$$= (x+5)(x+5) = (x+5)^2$$

$$x^2 - 14x + 49$$

$$= (x-7)(x-7) = (x-7)^2$$

$$x^2 + 3x + \frac{9}{4}$$

$$= \left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right) = \left(x+\frac{3}{2}\right)^2$$

E.g. Solve by completing the square.

$$x^2 + 4x - 1 = 0$$

$$x^2 + 4x + 4 = 1 + 4$$

$$(x+2)^2 = 5$$

$$x+2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

Solution set : $\{-2+\sqrt{5}, -2-\sqrt{5}\}$

Obj #4: Solving quadratic equations by using the quadratic formula.

$$ax^2 + bx + c = 0.$$

Quadratic formula :

The solutions are given by the formula:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Eg. Solve $2x^2 + 2x - 1 = 0$ using the quadratic formula.

$a = 2 \qquad b = 2 \qquad c = -1$

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 2 \cdot (-1)}}{4}$$

$$= \frac{-2 \pm \sqrt{4 + 8}}{4} = \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-1 \pm \sqrt{3}}{2}$$

E.g. Solve: $x^2 - 2x + 2 = 0$

$a = 1$; $b = -2$ c; 2

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2}$$

$$= \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i$$

Has no real solutions

But it has complex solutions: $\{1+i, 1-i\}$

The quantity b^2-4ac tells us the number of solutions and the type of solutions of the quadratic equation.

This quantity is called the discriminant of the equation.

1. If $b^2-4ac > 0$, then the equation has 2 real solutions.

2. If $b^2-4ac < 0$, then the equation has NO real solutions. It has 2 non-real solutions.

3. If $b^2-4ac = 0$, then the equation

has exactly 1 solution.