1.6. Other Types of Equations

Tuesday, September 12, 2017 9:54 AM

Objective #1: Solve polynomial equations by factoring.

E.g. $3x^4 - 48x^2 = 0$

$$3x^2 \cdot (x^2 - 16) = 0$$

$$3x^2 \cdot (x-4)(x+4) = 0$$

$3x^2 = 0$ or $x - 4 = 0$ or $x + 4 = 0$

$x^2 = 0$

$\boxed{x = 0}$ $\boxed{x = 4}$ $\boxed{x = -4}$

Solution set is $\{0, 4, -4\}$

E.g. Solve for x.

$$3x^3 + 2x^2 = 12x + 8$$

$$3x^3 + 2x^2 - 12x - 8 = 0$$

$$x^2(3x + 2) - 4(3x+2) = 0$$

$$(3x+2)(x^2 - 4) = 0$$

$$(3x+2)\cdot(x-2)\cdot(x+2) = 0$$

$3x+2 = 0$ or $x-2 = 0$ or $x+2 = 0$

$x = -\frac{2}{3}$ or $x = 2$ or $x = -2$

Solution set $\left\{-\frac{2}{3}, -2, 2\right\}$

Objective #2: Solve radical equations

E.g. $\sqrt{2x+13} = x+7$.

Square both sides:

$$\boxed{\left(\sqrt{2x+13}\right)^2 = (x+7)^2}$$

$$2x+13 = x^2+14x+49$$

$$0 = x^2+12x+36$$

$$0 = (x+6)^2$$

$$0 = x+6$$

$$x = -6.$$

Check solution by plugging $x = -6$ into the original equation:

$$\sqrt{2x+13} \stackrel{?}{=} x+7$$

$$\sqrt{1} \stackrel{?}{=} 1 \checkmark$$

Solution set: $\{-6\}$

E.g. $\sqrt{x+3} + 3 = x$

$$\sqrt{x+3} = x-3$$

Square both sides

$$\left(\sqrt{x+3}\right)^2 = (x-3)^2$$

$$x+3 = x^2 - 6x + 9$$

$$0 = x^2 - 7x + 6$$

$$x^2 - 7x + 6 = 0$$

$$(x-1)(x-6) = 0$$

$x = 1$ or $x = 6$

Check solutions:

Check $\cancel{\boxed{x = 1}}$ $\quad \sqrt{x+3} + 3 \stackrel{?}{=} x$

$$\sqrt{4} + 3 \stackrel{?}{=} 1$$

Extraneous Solution

$$5 \stackrel{?}{=} 1 \text{ Not equal.}$$

check $x = 6$ $\quad \sqrt{x+3} + 3 = x$

$$\sqrt{6+3} + 3 \stackrel{?}{=} 6$$

$$\sqrt{9} + 3 = 6 \checkmark$$

Solution Set $= \{6\}$.

Objective #3: Solve equations with Rational Exponents.

Quick Review of Rational Exponents.

$$(4)^{\frac{3}{2}}$$

$$\left(\sqrt{4}\right)^3 = (2)^3 = 8$$

$$\sqrt{(4)^3} = \sqrt{64} = 8$$

$$(8)^{\frac{5}{3}} = \left(\sqrt[3]{8}\right)^5 = (2)^5 = 32$$

In general,

$$a^{\frac{m}{n}}$$

$$\left(\sqrt[n]{a}\right)^m$$

$$\sqrt[n]{a^m}$$

E.g. Solve for x

$$5\cdot x^{\frac{3}{2}} - 25 = 0$$

$$5\cdot x^{\frac{3}{2}} = 25$$

$$x^{\frac{3}{2}} = 5$$

$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (5)^{\frac{2}{3}}$$

$$x = (5)^{\frac{2}{3}} = \left(\sqrt[3]{5}\right)^2 = \boxed{\sqrt[3]{25}}$$

Solution Set.

Check answer: $x = (5)^{\frac{2}{3}}$

$$5x^{\frac{3}{2}} - 25 \overset{?}{=} 0$$

$$5\cdot\left((5)^{\frac{2}{3}}\right)^{\frac{3}{2}} - 25 \overset{?}{=} 0$$

$$5\cdot 5 - 25 \overset{?}{=} 0 \quad \checkmark$$

E.g. Solve: $x^{\frac{2}{3}} - 8 = -4$

$$x^{\frac{2}{3}} = 4$$

$$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(4)^{\frac{3}{2}}$$

$$x = \pm\left(\sqrt{4}\right)^{3}$$

$$x = \pm 8$$

Check answers $\longrightarrow$ Do it.

Solution Set: $\{8, -8\}$.

Objective #4: Solve equations quadratic in form.

E.g. Solve: $x^4 - 5x^2 + 4 = 0$

Make a substitution.

let $t = x^2$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4$ or $t = 1$

$x^2 = 4$ or $x^2 = 1$

$x = \pm 2$ or $x = \pm 1$

Solution Set: $\{2, -2, 1, -1\}$

E.g. $3x^{\frac{2}{3}} - 11x^{\frac{1}{3}} - 4 = 0$

let $t = x^{\frac{1}{3}}$

$$3t^2 - 11t - 4 = 0$$

$$(3t + 1)(t - 4) = 0$$

$$3t + 1 = 0 \quad \text{or} \quad t - 4 = 0$$

$$t = -\frac{1}{3} \quad \text{or} \quad t = 4$$

$$x^{\frac{1}{3}} = \quad \text{or} \quad x^{\frac{1}{3}} = 4$$

$$\left(x^{\frac{1}{3}}\right)^3 = \left(-\frac{1}{3}\right)^3 \quad \text{or} \quad \left(x^{\frac{1}{3}}\right)^3 = (4)^3$$

$$x = -\frac{1}{27} \quad \text{or} \quad x = 64$$

Solution set $\left\{-\frac{1}{27}, 64\right\}$.

Objective #5: Solve equations that involve absolute value.

E.g Solve $|x-2| = 7$

$x-2=7$ or $x-2=-7$

$\boxed{x=9}$ or $\boxed{x=-5}$

Key: $|\text{Stuff}| = a$

Suppose $a>0$

Stuff $= a$ or Stuff $= -a$.

E.g. $|2x-1|+2=7$

$|2x-1| = 5$

$2x - 1 = 5$ or $2x - 1 = -5$

$\boxed{x = 3}$ or $\boxed{x = -2}$

An application problem.

The formula $t = \frac{\sqrt{d}}{2}$ models a basket ball player's hang time t in seconds in terms of the vertical distance d, in feet. If the hang time is 1.6 seconds, find the vertical distance d?

$$1.6 = \frac{\sqrt{d}}{2}$$

$$3.2 = \sqrt{d}$$

$$d = (3.2)^2 = 10.24 \text{ ft.}$$