2.2. More on Functions and their graphs
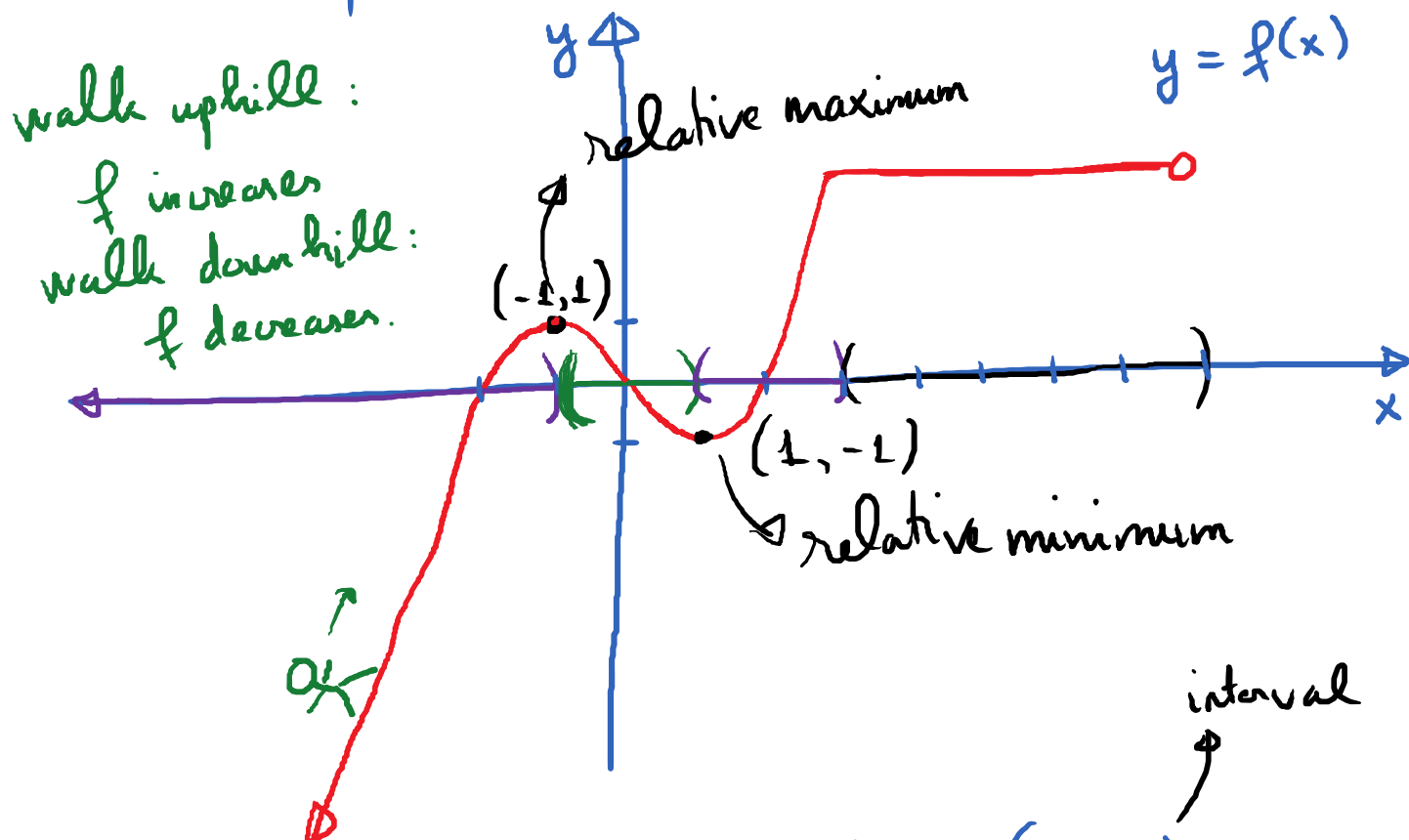
Objective #1: Identify the intervals on which a function increases, decreases or constant

walk uphill :
  f increases
walk down hill :
  f decreases.

relative maximum

$y = f(x)$

$(-1, 1)$

$(1, -1)$

relative minimum

OK

interval

function increases on $(-\infty, -1) \cup (1, 3)$

function decreases on : $(-1, 1) \longrightarrow$ interval
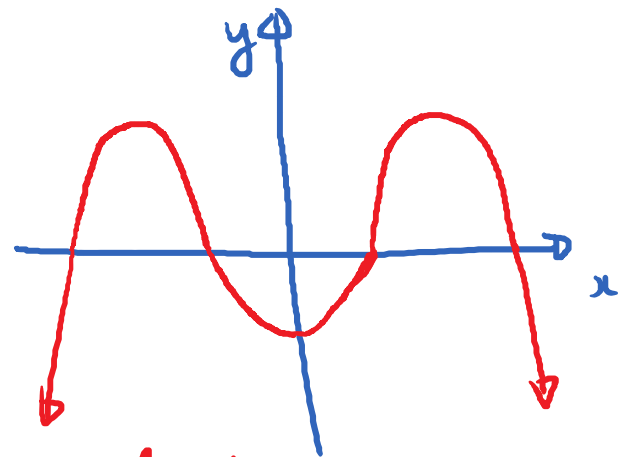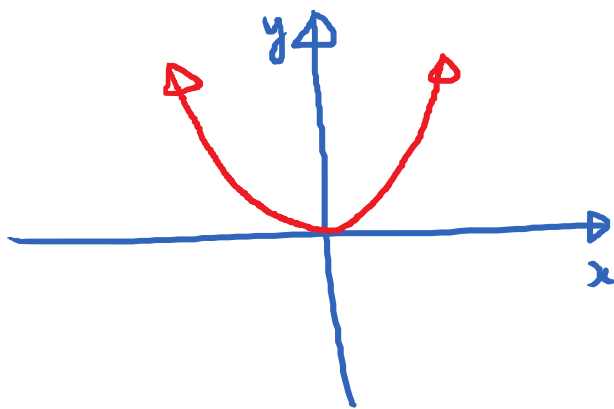
function is constant on: $(3, 8) \longrightarrow$ interval

Relative maximum is the point : $(-1, 1)$
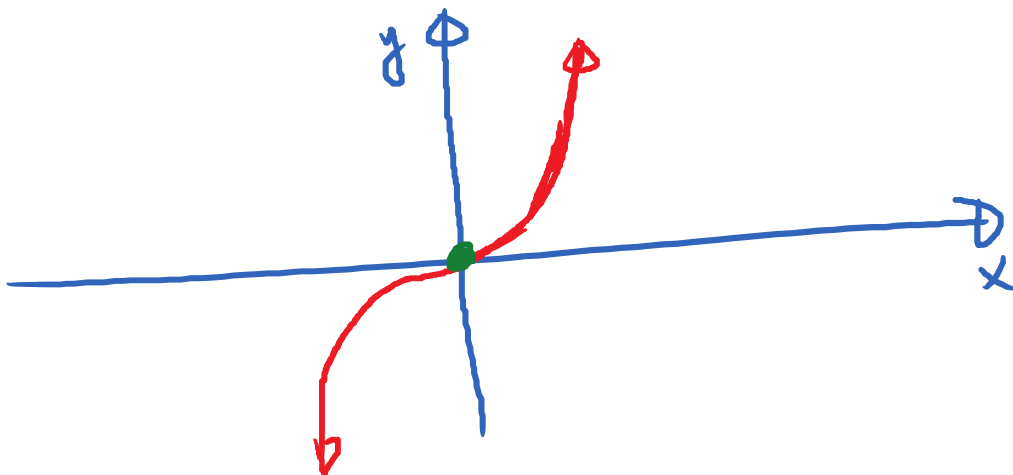
Relative minimum is the point : $(1, -1)$

Objective #2: Identify even and odd functions and their symmetries.

An even function is a function whose graph is symmetric with respect to the y-axis.

graphs of even functions

Odd functions are functions whose graphs are symmetric with respect to the origin (0,0)

How to determine whether a formula $y = f(x)$ corresponds to an even or an odd function.

Even: If $f(-x) = f(x)$ for all x in the domain of f, then f is an even function.

E.g. $f(x) = x^2 - x^4$

Even or odd?

$$f(-x) = (-x)^2 - (-x)^4$$
$$= x^2 - x^4 = f(x)$$

So, $f(-x) = f(x)$.

Hence, f is an even function.

<u>Odd</u>: If $f(-x) = -f(x)$ for all x in the domain of f, then f is an odd function.

E.g. $f(x) = x^3 + x$

$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x$$
$$= -\left(\boxed{x^3 + x} \right)$$
$$= -f(x)$$

So, $f(-x) = -f(x)$

Hence, the function is odd

E.g. $f(x) = x^5 + 6$.   Odd or even?

$$f(-x) = (-x)^5 + 6 = -x^5 + 6$$

This function is neither odd nor even.

Objective 3: Understand and use piecewise functions.

A function that is defined by two or more equations over a specified domain is called a piecewise function.

E.g.

$$f(x) = \begin{cases} 3x & \text{if } x \leq -1 \\ x-2 & \text{if } x > -1 \end{cases}$$

Evaluate $f(-10) = 3 \cdot (-10) = -30$

$$f(-1) = 3 \cdot (-1) = -3$$

$$f(0) = 0 - 2 = -2$$

$$f(2017) = 2017 - 2 =$$
$$2015$$