

E.g. Given that :

$$C(t) = \begin{cases} 20 & \text{if } 0 \leq t \leq 60 \\ 20 + 0.4(t - 60) & \text{if } 60 < t < 1000 \\ 20000 & \text{if } t \geq 1000 \end{cases}$$

Evaluate $C(40) = 20$

$$\begin{aligned} C(70) &= 20 + 0.4(70 - 60) \\ &= 20 + (0.4) \cdot 10 = 24 \end{aligned}$$

$$C(1001) = 20000$$

E.g. Graph a piecewise function.

$$f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ x - 2 & \text{if } x > -1 \end{cases}$$

Graph the pieces of the function

| x | $f(x) = 3$ |
|-----|-------------------------|
| -1 | 3 $\rightarrow (-1, 3)$ |
| -2 | 3 $\rightarrow (-2, 3)$ |
| -3 | 3 $\rightarrow (-3, 3)$ |

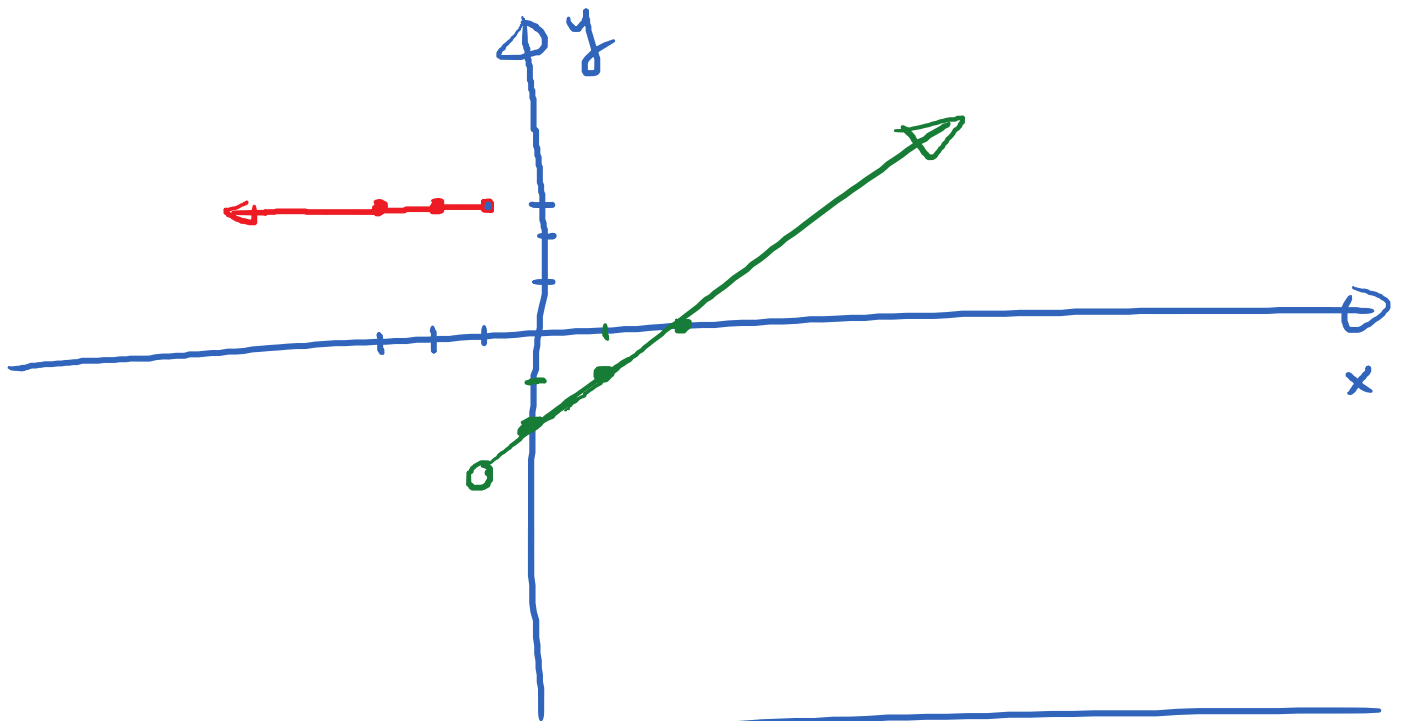
| x | $f(x) = x - 2$ |
|-----|--------------------------|
| 0 | -2 $\rightarrow (0, -2)$ |
| 1 | -1 $\rightarrow (1, -1)$ |
| 2 | 0 $\rightarrow (2, 0)$ |

$$\begin{array}{c|c} -2 & 3 \\ -3 & 3 \end{array} \rightarrow (-2, 3)$$

$$\begin{array}{c|c} -3 & 3 \end{array} \rightarrow (-3, 3)$$

$$\begin{array}{c|c} 1 & -1 \\ 2 & 0 \end{array} \rightarrow (1, -1)$$

$$\begin{array}{c|c} 2 & 0 \end{array} \rightarrow (2, 0)$$



Objective #4: Find and simplify the difference quotient of a function.

The expression: $\frac{f(x+h) - f(x)}{h}$, $h \neq 0$
is called the difference quotient of the function f .

E.g. $f(x) = 2x$. Find and simplify the difference quotient of f .

$$f(x+h) = 2(x+h); \quad f(x) = 2x$$

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{2(x+h) - 2x}{h} \\ &= \frac{\cancel{2x} + 2h - \cancel{2x}}{h} \\ &= \frac{2h}{h} = \boxed{2}\end{aligned}$$

E.g. $f(x) = x^2 - 4x + 3$.

Find and simplify the difference quotient for f .

$$f(x+h) = (x+h)^2 - 4(x+h) + 3$$

$$= x^2 + 2xh + h^2 - 4x - 4h + 3$$

$$f(x) = x^2 - 4x + 3$$

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{x^2 + 2xh + h^2 - 4x - 4h + 3 - (x^2 - 4x + 3)}{h} \\ &= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{3} - \cancel{x^2} + \cancel{4x} - \cancel{3}}{h}\end{aligned}$$

$$= \frac{2xh + h^2 - 4h}{h} = \frac{\cancel{h}(2x + h - 4)}{\cancel{h}}$$

$$= \boxed{2x + h - 4}$$