2.6 Combinations of Functions and Composite Functions

Tuesday, October 10, 2017 11:12 AM

Obj 1: Find the domain of a function.

Division by zero

Take a square root or even root of a negative #.

To find the domain of a function, we exclude from the domain the real values of x that cause division by zero or taking even root of a negative number.

E.g. Find the domain of $g(x) = \dfrac{5x}{x^2 - 49}$.

Step 1: $x^2 - 49 = 0$

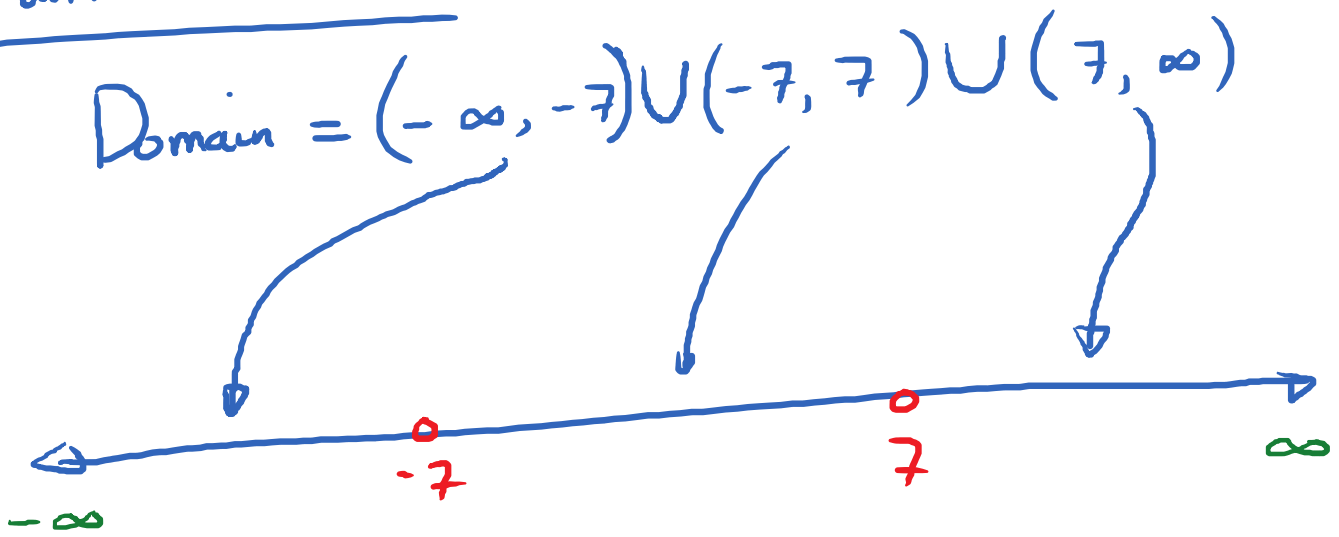
$x^2 = 49$

$x = \pm 7$

Step 2: Domain is all real numbers except for $x = 7$ and $x = -7$.

In interval notation:

Domain $= (-\infty, -7) \cup (-7, 7) \cup (7, \infty)$

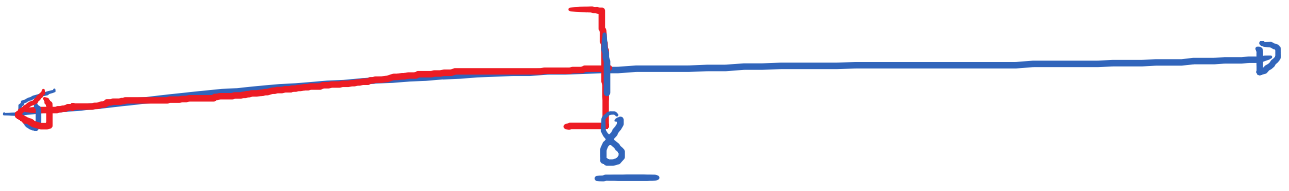


E.g. $g(x) = \sqrt{8 - 5x}$. Find domain of g.

To find domain: Require $8 - 5x \geq 0$

$8 \geq 5x$; $5x \leq 8$; $x \leq \frac{8}{5}$.

In interval notation: $(-\infty, \frac{8}{5}]$

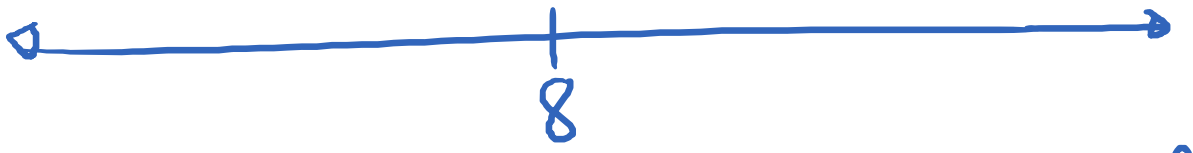


E.g. $h(x) = \frac{5x}{\sqrt{24 - 3x}}$. Find domain.

To find domain: $24 - 3x > 0$

$24 > 3x$; $8 > x$

Interval: $(-\infty, 8)$

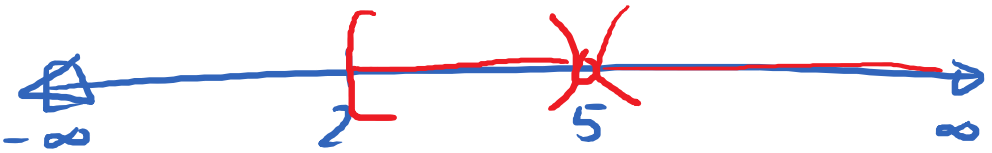


E.g. $u(x) = \dfrac{\sqrt{x-2}}{x-5}$. Find the domain of u.

$x - 2 \geq 0 \rightarrow \boxed{x \geq 2}$

$\boxed{x \neq 5}$

Interval Notation:



Domain: $[2, 5) \cup (5, \infty)$

Obj 2: Algebra of functions

f, g: functions.

$f + g, \; f - g, \; f \cdot g, \; \dfrac{f}{g}$

$f+g$, $f-g$, $f \cdot g$, $\frac{f}{g}$

E.g. $f(x) = x - 5$; $g(x) = x^2 - 1$.

* Find $(f+g)(x)$

$$(f+g)(x) = f(x) + g(x)$$
$$= (x-5) + (x^2-1)$$
$$(f+g)(x) = x^2 + x - 6$$

* Find $(f+g)(-1) = (-1)^2 + (-1) - 6 = 1 - 1 - 6$

$$(f+g)(-1) = -6$$

$f(-1) = -6$; $g(-1) = (-1)^2 - 1 = 0$

$$f(-1) + g(-1) = -6 + 0 = -6$$

* Find $(f-g)(x)$

$$(f-g)(x) = f(x) - g(x)$$
$$= (x-5) - (x^2-1)$$
$$= x - 5 - x^2 + 1$$
$$(f-g)(x) = -x^2 + x - 4$$

* Find $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$
$$= (x-5)\cdot(x^2-1)$$
$$= x^3 - x - 5x^2 + 5$$
$$(fg)(x) = x^3 - 5x^2 - x + 5$$

* $(fg)(-1) = (-1)^3 - 5(-1)^2 - (-1) + 5$
$= -1 - 5\cdot 1 + 1 + 5 = 0$

* Find $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x-5}{x^2-1}.$$

Domain = ? $x^2 - 1 = 0$; $x^2 = 1$; $x = \pm 1$.

Domain: all real numbers except for $x = 1$ and $x = -1$.

Interval: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$