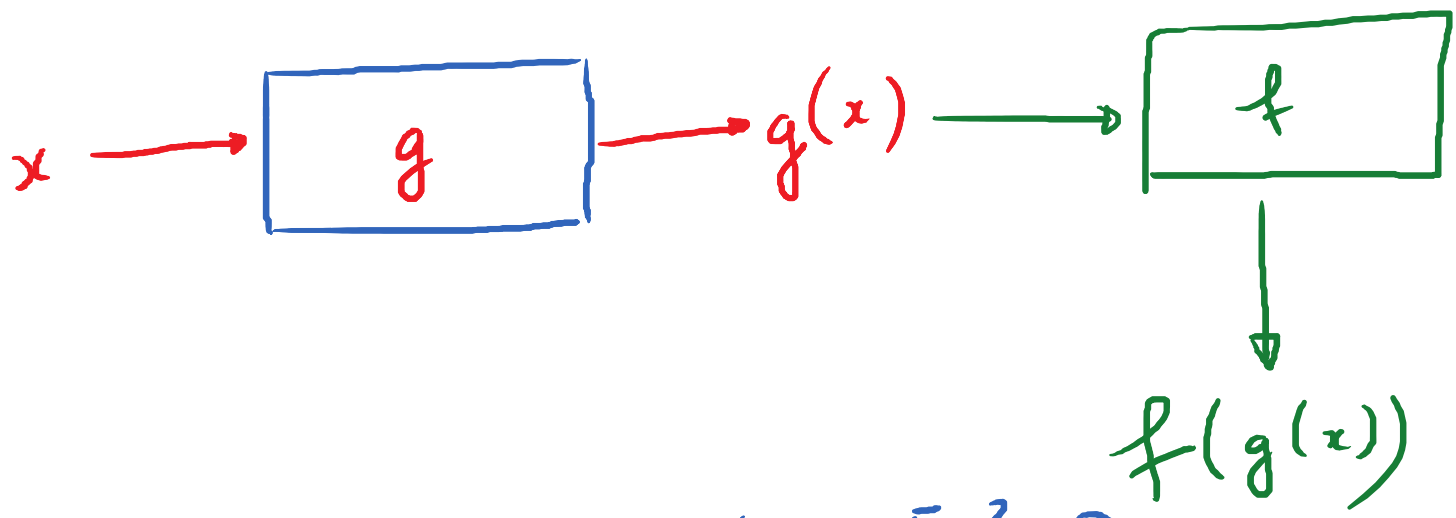
Obj 3: Composition of functions.

The composition of the function f with the function g is a function, denoted by $f \circ g$ (read as f circle g or f of g)

$$(f \circ g)(x) = f(g(x))$$

Note: $f(g(x))$ does NOT mean $(fg)(x)$



E.g. $f(x) = 4x - 3$. $g(x) = 5x^2 - 2$.

(a) Find $(f \circ g)(x)$

$$(f \circ g)(x) = f(g(x))$$
$$= f(5x^2 - 2)$$
$$= 4\cdot(5x^2 - 2) - 3$$
$$= 20x^2 - 8 - 3$$
$$\boxed{(f \circ g)(x) = 20x^2 - 11}$$

(b) Evaluate $(f \circ g)(2) = 20 \cdot 4 - 11 = 69$

(c) Find $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x)) = g(4x - 3)$$
$$= 5(4x - 3)^2 - 2$$
$$= 5\cdot(16x^2 - 24x + 9) - 2$$
$$= 80x^2 - 120x + 45 - 2$$
$$\boxed{(g \circ f)(x) = 80x^2 - 120x + 43}$$

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Note: In general, $(f\circ g)(x) \neq (g\circ f)(x)$

Ex.

(a) $f(x) = 2x+1$. $g(x) = 2x^2 - x - 1$.

Find $(f\circ g)(x)$ and $(g\circ f)(x)$

(b) $f(x) = \dfrac{4}{x+2}$ and $g(x) = \dfrac{1}{x}$.

Find $(f\circ g)(x)$ and $(g\circ f)(x)$

(a) $(f\circ g)(x) = f(g(x)) = f(2x^2 - x - 1)$

$= 2(2x^2 - x - 1) + 1$

$= 4x^2 - 2x - 2 + 1$

$(f\circ g)(x) = 4x^2 - 2x - 1$

$(g\circ f)(x) = g(f(x)) = g(2x+1)$

$= 2(2x+1)^2 - (2x+1) - 1$

$= 2(4x^2+4x+1) - 2x - 1 - 1$

$(g\circ f)(x) = 8x^2 + 8x + 2 - 2x - 2 = 8x^2 + 6x$

(b) $(f\circ g)(x) = f(g(x)) = \dfrac{4}{\frac{1}{x}+2} = \dfrac{4}{\frac{1}{x}+\frac{2x}{x}}$

$= \dfrac{4}{\frac{1+2x}{x}} = 4\cdot\dfrac{x}{1+2x} = \dfrac{4x}{1+2x}.$

$(g\circ f)(x) = g(f(x)) = \dfrac{1}{\frac{4}{x+2}} = \dfrac{x+2}{4}.$

Obj 4: Find the domain of Composite Functions.

E.g. $f(x) = \dfrac{4}{x+2}$; $g(x) = \dfrac{1}{x}$.

We saw that $(f\circ g)(x) = f(g(x)) = \dfrac{4x}{1+2x}$.

(1) Find Domain of $g(x)$. $\boxed{x \neq 0}$.

(2) Find Domain of $(f\circ g)(x)$. $(f\circ g)(x) = \dfrac{4x}{1+2x}$

$1+2x=0 \rightarrow x = -\frac{1}{2}$. Domain : $\boxed{x \neq -\frac{1}{2}}$.

Domain of $f \circ g$:

$$\left(-\infty, -\tfrac{1}{2}\right) \cup \left(-\tfrac{1}{2}, 0\right) \cup (0, \infty)$$

To find domain of the composite function $f \circ g$:

1. Find domain of g.
2. Form $(f \circ g)(x)$.
3. Find domain of $f(g(x))$
4. Take the intersection of domain in 1 and 3

E.g. $f(x) = \frac{2}{x+3}$. $g(x) = \frac{1}{x-1}$.

* Find domain of $f \circ g$.

1. Domain of $g(x)$: $x \neq 1$.

2. Find $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x-1}\right)$

$$= \frac{2}{\frac{1}{x-1} + 3} = \frac{2}{\frac{1}{x-1} + \frac{3(x-1)}{x-1}}$$

$$(f \circ g)(x) = \frac{2}{\frac{3x-2}{x-1}} = \frac{2(x-1)}{3x-2}$$

3. Domain of $f(g(x))$: $x \neq \frac{2}{3}$.

4. Domain of $f \circ g$: $(-\infty, \frac{2}{3}) \cup (\frac{2}{3}, 1) \cup (1, \infty)$

Obj 5. Decompose Functions.

E.g. $h(x) = \sqrt{x^2 + 5}$.

Find functions $f(x)$ and $g(x)$ (none which is the identity function) such that

$$(f \circ g)(x) = h(x)$$

$$\boxed{f(x) = \sqrt{x} \;;\; g(x) = x^2 + 5}$$

$$(f \circ g)(x) = \sqrt{x^2+5} = h(x) \checkmark$$

$$\boxed{f(x) = \sqrt{x+5} \;,\; g(x) = x^2}$$

$$(f \circ g)(x) = \sqrt{x^2+5} = h(x) \checkmark$$

E.g. $h(x) = (3x-1)^5$.

Find f and g s.t. $(f \circ g)(x) = h(x)$

$f(x) = x^5$; $g(x) = 3x - 1$