2.7. Inverse Functions.

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E.g. $f(x) = 4x - 7$

$$g(x) = \frac{x+7}{4}$$

$$f \circ g(x) = f(g(x)) = f\left(\frac{x+7}{4}\right)$$

$$= \cancel{4} \cdot \left(\frac{x+7}{\cancel{4}}\right) - 7$$

$$= x + 7 - 7 = x$$

$$f \circ g(x) = f(g(x)) = x$$

$$g \circ f(x) = g(f(x)) = g(4x - 7)$$

$$= \frac{4x - 7 + 7}{4} = \frac{4x}{x} = x.$$

$$g \circ f(x) = g(f(x)) = x$$

Obj 1: Define the inverse of a function and verify that 2 functions are inverses.

Let f and g be functions such that

$f(g(x)) = x$ for every x in the domain of g

$g(f(x)) = x$ for every x in the domain of f

Then we say that g is the inverse of the function f and we denote the function g as f^{-1}.

So, $f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$.

E.g. Verify Inverse functions

Determine whether $f(x) = \dfrac{3}{x-4}$ and $g(x) = \dfrac{3}{x} + 4$ are inverses of each other.

$$f(g(x)) = f\left(\frac{3}{x}+4\right) = \frac{3}{\left(\frac{3}{x}+4\right)-4}$$

$$= \frac{3}{\frac{3}{x}} = 3\cdot\frac{x}{3} = \frac{\cancel{3}x}{\cancel{3}} = x$$

$\rightarrow f(g(x)) = x$

$$g(f(x)) = g\left(\frac{3}{x-4}\right) = \frac{3}{\frac{3}{x-4}} + 4$$

$$= \cancel{3}\cdot\frac{(x-4)}{\cancel{3}} + 4$$

$$= x-4+4 = x$$

$\rightarrow g(f(x)) = x$

Therefore f and g are inverses of each other.

So we write $f^{-1}(x) = \frac{3}{x}+4$.

Note: The notation $f^{-1}(x)$ $\boxed{\text{does not}}$ mean $\frac{1}{f(x)}$. It means the inverse function of f.

Obj 2: Method to find the inverse function of a given function.

(1) Replace the notation $f(x)$ by the letter y in the equation for $f(x)$.

(2) Solve for x in terms of y. (Get x by itself)

(3) Interchange the letter x and the letter y in the equation of step (2)

(4) Replace the letter y in step (3) by the notation $f^{-1}(x)$.

E.g. Apply the method outlined above to find the inverse of $f(x) = 4x^3 - 1$.

1. $y = 4x^3 - 1$.

2. $y + 1 = 4x^3$

$$\frac{y+1}{4} = x^3$$

$$\sqrt[3]{\frac{y+1}{4}} = x$$

$$x = \sqrt[3]{\frac{y+1}{4}}$$

3. $y = \sqrt[3]{\frac{x+1}{4}}$

4. $\boxed{f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}}$

E.g. Find the inverse of $f(x) = \frac{5}{x} - 6$

1. $y = \frac{5}{x} - 6$

2. $x(y+6) = \left(\frac{5}{\cancel{x}}\right) \cdot \cancel{x} \rightarrow x \cdot (y+6) = 5$

$\rightarrow x = \frac{5}{y+6}$

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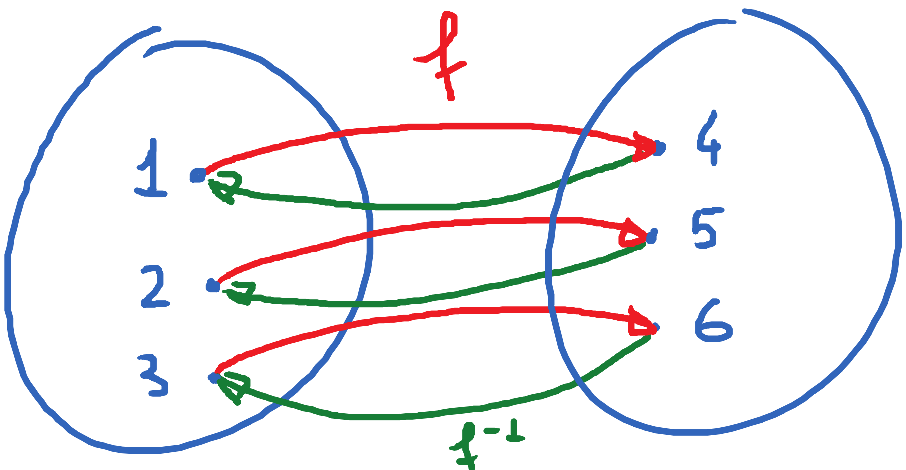
(3) $y = \dfrac{5}{x+6}$.

(4) $\boxed{f^{-1}(x) = \dfrac{5}{x+6}.}$

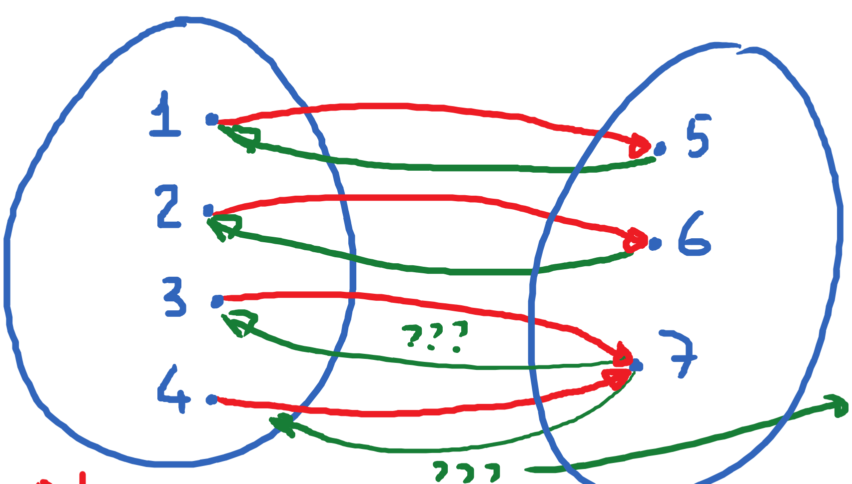
<u>Obj 3</u>: Horizontal line test to determine whether a graph is the graph of a one-to-one function.



f has an inverse.

One-to-one

Every output corresponds to exactly 1 input



f^{-1} get confused

f NOT one-to-one. Output 7 has 2 inputs

f NOT one-to-one. Output 7 has 2 inputs