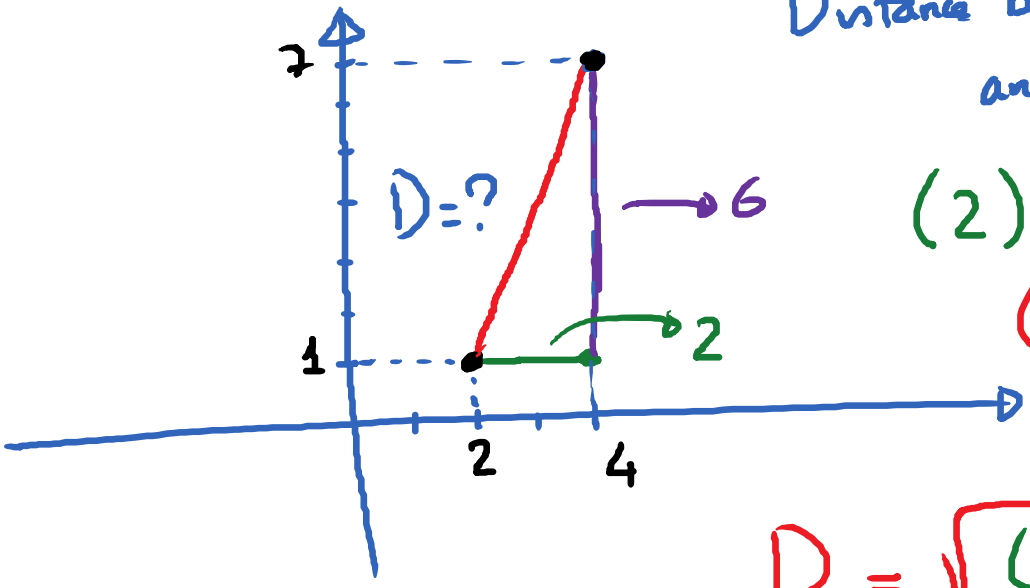
2.8. Distance and Midpoint Formula, Circles

Tuesday, October 17, 2017 11:09 AM

Obj 1: Distance Formula.



Distance between (2,1) and (4,7)

$(2)^2 + (6)^2 = D^2$

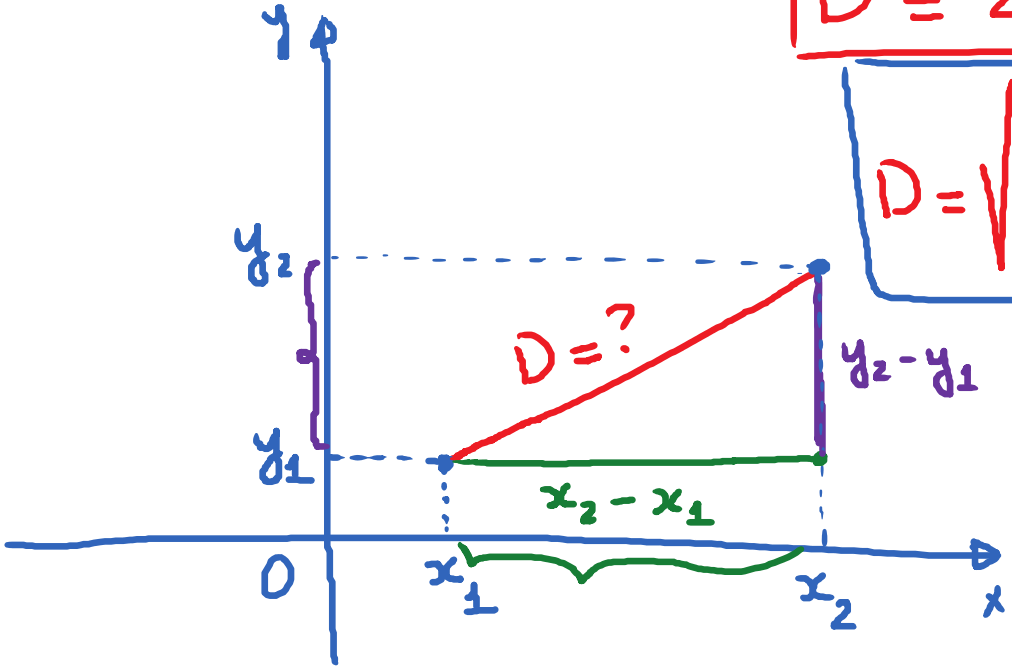
(by Pythagorean Theorem)

$D = \sqrt{(2)^2 + (6)^2}$

$D = \sqrt{4 + 36}$

$D = \sqrt{40} = \sqrt{4 \cdot 10}$

$\boxed{D = 2\sqrt{10}}$



$$\boxed{D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

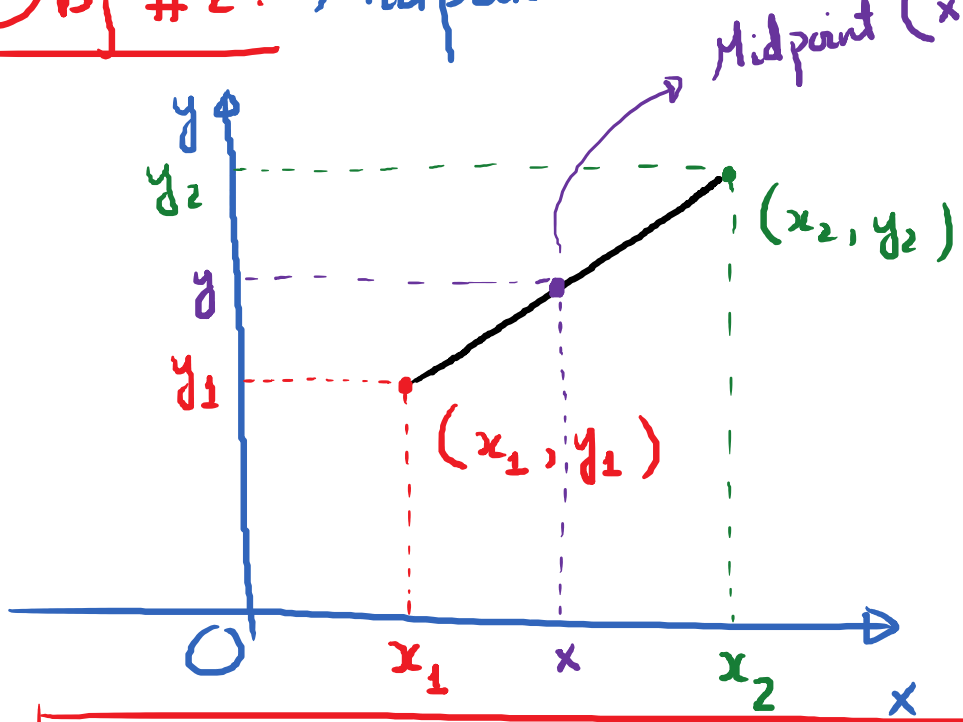
E.g. Find the distance between $(-1,-3)$ and $(2,3)$. Simplify your answer as much as possible.

$$D = \sqrt{\left(2-(-1)\right)^2 + \left(3-(-3)\right)^2}$$

$$= \sqrt{9+36} = \sqrt{45} = \sqrt{9\cdot 5} = \sqrt{9}\cdot\sqrt{5}$$

$$\boxed{D = 3\sqrt{5}}$$

Obj #2: Midpoint Formula



$$x_{midpoint} = \frac{x_1 + x_2}{2} \; ; \; y_{midpoint} = \frac{y_1 + y_2}{2}$$

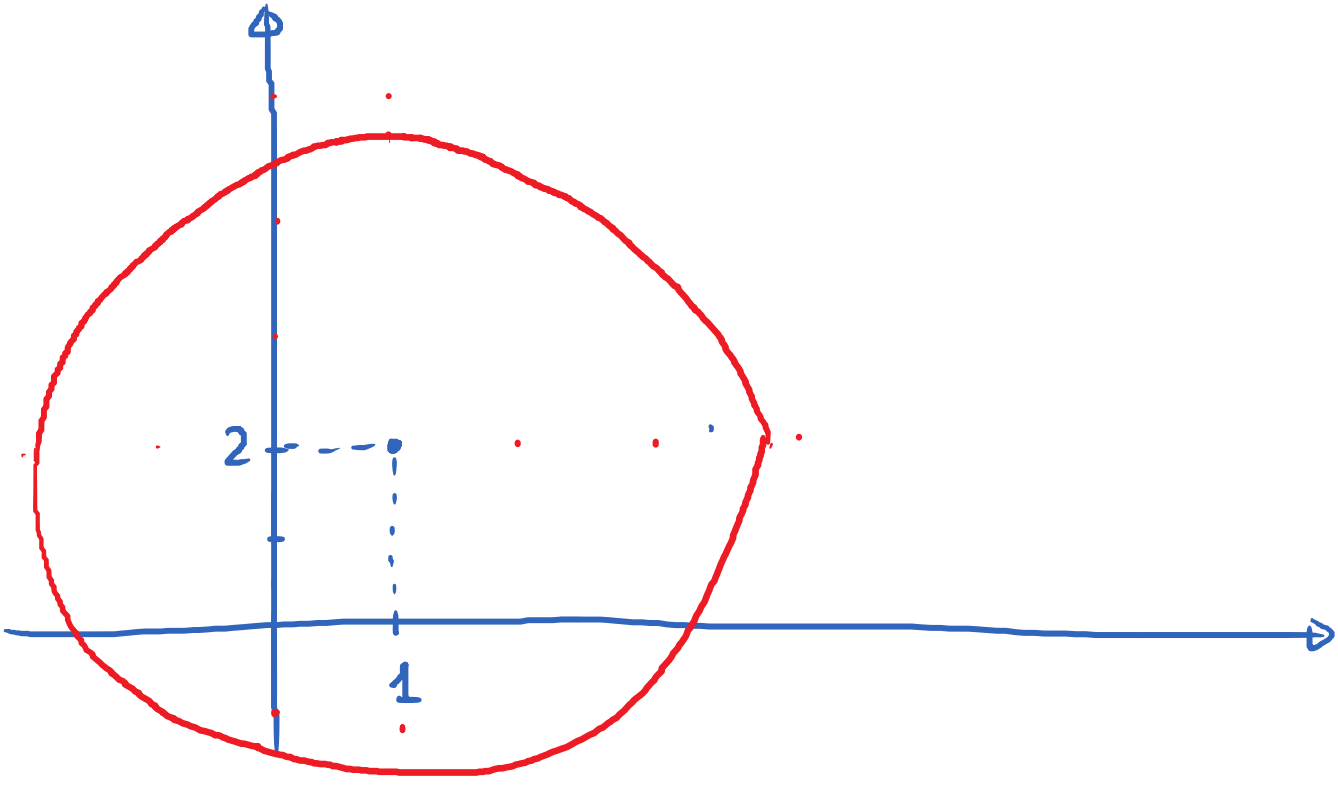
E.g. Find the midpoint of the line segment with endpoints $(1,2)$ and $(7,-3)$.

$$x_{midpoint} = \frac{1+7}{2} = 4$$

$$y_{midpoint} = \frac{2-3}{2} = -\frac{1}{2}$$

} Midpoint $\left(4, -\frac{1}{2}\right)$

Obj 3: The Standard Equation of a circle.



Center $= (1,2)$. Radius $= 3$

(x, y) is any point on the circle, then:

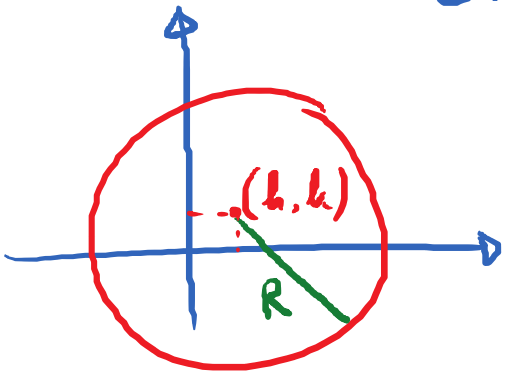
distance from (x, y) to $(1, 2)$ $= 3$

$$\sqrt{(x-1)^2 + (y-2)^2} = 3$$

$$\boxed{(x-1)^2 + (y-2)^2 = 9}$$

↳ the standard equation of the circle with center $(1, 2)$ and radius 3

In general, the standard equation for the circle with center (h, k) and Radius R is:



$$\boxed{(x-h)^2 + (y-k)^2 = R^2}$$

E.g. Find Standard equation for the circle with center $(-4, -8)$ and radius $= 6$.

$$(x+4)^2 + (y+8)^2 = 36$$

E.g. Given the standard equation of a circle:

$$(x+5)^2 + (y-11)^2 = 5$$

Find center and radius of this circle.

Center: $(-5, 11)$. Radius: $\sqrt{5}$

Obj #4: The general form of the equation of a circle.

E.g. $x^2 + y^2 + 4x - 4y - 1 = 0$ ← general equation of a circle

Complete the square:

$$x^2 + 4x + 4 + y^2 - 4y + 4 = 1 + 8$$

Standard equation:

$$(x+2)^2 + (y-2)^2 = 9$$

→ Center: $(-2, 2)$; Radius $= 3$

Ex. Write the given general equation of the circle in standard form.

Find the center and radius and graph the circle.

$$x^2 + y^2 + 8x - 2y - 8 = 0$$

$$x^2 + 8x + y^2 - 2y = 8$$

$$x^2 + 8x + 16 + y^2 - 2y + 1 = 8 + 17$$

$$(x+4)^2 + (y-1)^2 = 25$$

Center: $(-4, 1)$. Radius 5.