* How to find zeros of a polynomial Function.

E.g. Find all zeros (roots) of

$$f(x) = x^3 + 2x^2 - 4x - 8$$

To find the zeros for f, we first set $f(x) = 0$.

$$x^3 + 2x^2 - 4x - 8 = 0$$

We want to solve for x.

$$x^2\boxed{(x+2)} - 4\boxed{(x+2)} = 0$$

$$(x+2)(x^2-4) = 0$$

$$(x+2)(x+2)(x-2) = 0$$

$$(x+2)^2(x-2) = 0$$

$(x+2)^2 = 0$ or $(x-2) = 0$

multiplicity 2

multiplicity 1

$x + 2 = 0$

$\boxed{x = -2}$ $\boxed{x = 2}$

The zeros of f are: $x = -2$, $x = 2$

$x = 2$

We say that $x = -2$ is a zero with multiplicity 2.

$x = 2$ is a zero with multiplicity 1.

E.g. Find the zeros and the multiplicity for each zero of the function

$$f(x) = \left(x - \frac{1}{2}\right)^3 \cdot (2x+1)^2 \cdot (3x-4)$$

Set $f(x) = 0$. Then

$\left(x - \frac{1}{2}\right)^3 = 0$ or $(2x+1)^2 = 0$ or $3x - 4 = 0$

$x - \frac{1}{2} = 0$

$\boxed{x = \frac{1}{2}}$

Multiplicity $= 3$

$2x + 1 = 0$

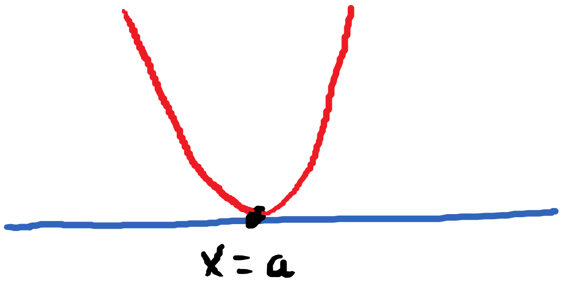
$\boxed{x = -\frac{1}{2}}$

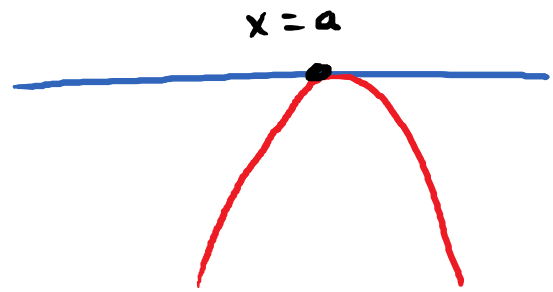
Multiplicity $= 2$

$\boxed{x = \frac{4}{3}}$

Multiplicity $= 1$

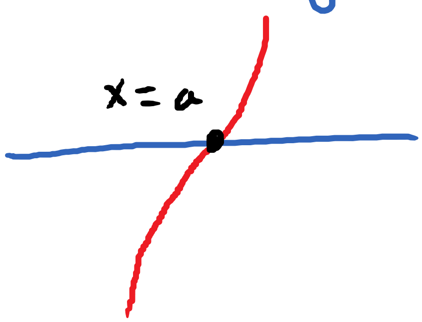
Why do we care about multiplicity?

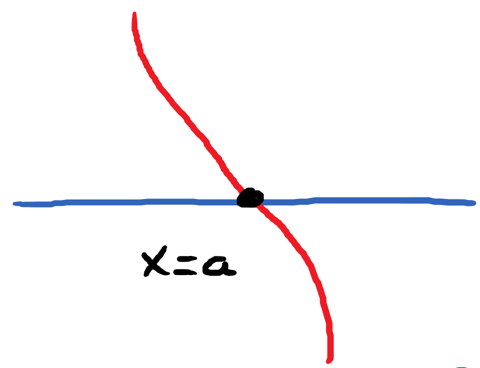




$x = a$ is a zero with EVEN multiplicity.

graph bounces back.





$x = a$ is a zero with ODD multiplicity.

graph crosses the x-axis at that point

Obj 4: Strategy for graphing polynomial functions.

1 Determine the end behavior.

* Find the leading term and apply the leading term test.

2. Find zeros (x-parts of x-intercepts)

   Set $f(x) = 0$ and try to factor.

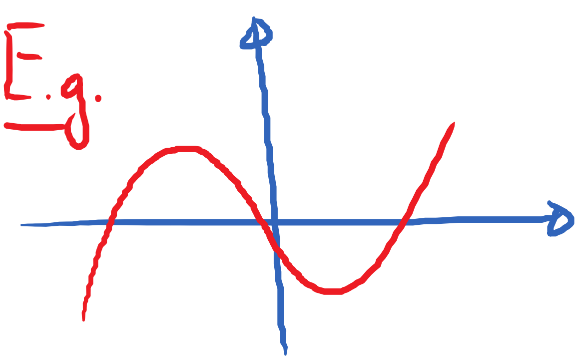
3. Find the multiplicity of zeros to determine the behavior of the graph at those x-intercepts.

4. Find y-intercept.

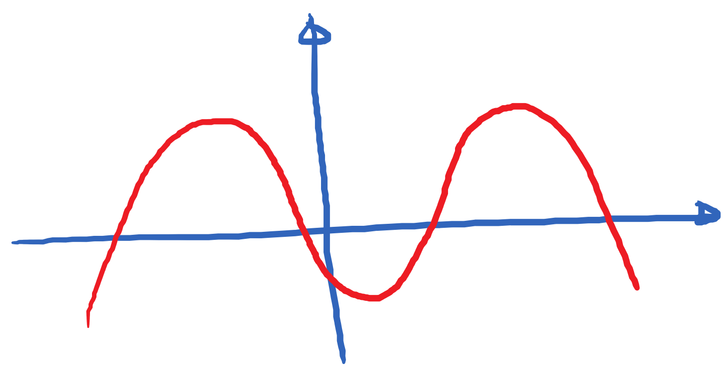
   To find y-intercept, we set $x = 0$ in the formula of the function.

5. As a quick check we can count the # of turning points and compare it with the degree.

E.g.



2 turning points



3 turning points

As a general rule, the # of turning points can not exceed $n-1$ where n is the degree of the polynomial.

E.g. Graph a Polynomial Function.

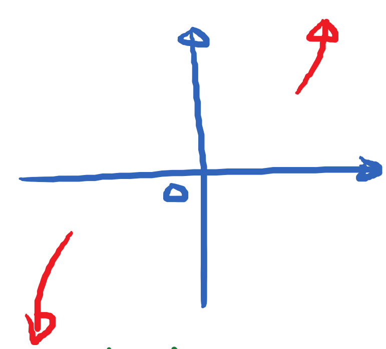
Use the strategy described above to graph the polynomial function

$$f(x) = 2(x+2)^2(x-3)$$

Step 1: Determine the end behavior.

leading term: $2x^3$

$\longrightarrow$ End Behavior: Rises to the right
Falls to the left



Step 2: Find zeros (x-parts of x-intercepts)

Set $f(x) = 0$.

$$2(x+2)^2(x-3) = 0$$

So, $(x+2)^2 = 0$ or $x - 3 = 0$

$x + 2 = 0 \qquad x = 3$

$x = -2$

x-intercepts: $(-2, 0)$; $(3, 0)$.

Step 3: Determine the multiplicity of each zero.

$x = -2$ has multiplicity 2.

$x = 3$ has multiplicity 1.

Step 4: Find y-intercept.

Set $x = 0$ in the formula for f.

$$f(x) = 2(x+2)^2(x-3)$$

$$f(0) = 2\cdot(2)^2\cdot(-3) = 2\cdot 4\cdot(-3) = -24.$$

y-intercept: $(0, -24)$.

graph of $f(x) = 2(x+2)^2(x-3)$