3.3-Dividing Polynomials - Remainder and Factor Theorems

Thursday, November 2, 2017 10:13 AM

Obj 1: Use Long Division to Divide Polynomial.

Dividend

E.g. Divide $\boxed{2x^4 + 3x^3 - 7x - 10}$ by $\boxed{x^2 - 2x}$ Divisor

quotient

$$\frac{2x^4}{x^2} = 2x^2 \qquad \frac{7x^3}{x^2} = 7x \qquad \frac{14x^2}{x^2} = 14$$

$$
\begin{array}{r|l}
 & 2x^2 + 7x + 14 \\
\hline
\boxed{x^2} - 2x & \boxed{2x^4} + 3x^3 + 0x^2 - 7x - 10 \\
 & -(2x^4 - 4x^3) \\
\hline
 & 0 + \boxed{7x^3} + 0x^2 - 7x - 10 \\
 & -(7x^3 - 14x^2) \\
\hline
 & \boxed{14x^2} - 7x - 10 \\
 & -(14x^2 - 28x) \\
\hline
 & 21x - 10
\end{array}
$$

Remainder.

degree = 1 < degree of divisor

→ process ends.

Result of dividing $2x^4 + 3x^3 - 7x - 10$ by $x^2 - 2x$

is : Quotient $= 2x^2 + 7x + 14$

Remainder $= 21x - 10$.

$$\frac{\overbrace{\boxed{2x^4 + 3x^3 - 7x - 10}}^{\text{Dividend}}}{\underbrace{\boxed{x^2 - 2x}}_{\text{Divisor}}} = \overbrace{\boxed{2x^2 + 7x + 14}}^{\text{quotient}} + \frac{\overbrace{\boxed{21x - 10}}^{\text{Remainder}}}{x^2 - 2x}$$

Another way to write the result is :

$$\underbrace{2x^4 + 3x^3 - 7x - 10}_{\text{Dividend}} = \underbrace{(x^2 - 2x)}_{\text{Divisor}} \cdot \underbrace{(2x^2 + 7x + 14)}_{\text{Quotient}} + \underbrace{(21x - 10)}_{\text{Remainder}}$$

Dividend = Divisor · Quotient + Remainder

Ex. Use long Division to divide

$7 - 11x - 3x^2 + 2x^3$ by $x - 3$

Done in class :

$$\frac{2x^3 - 3x^2 - 11x + 7}{x - 3} = \overbrace{\boxed{2x^2 + 3x - 2}}^{\text{quotient}} + \frac{\overbrace{\boxed{1}}^{\text{Remainder.}}}{x - 3}$$

Thursday, November 2, 2017 10:38 AM

Obj 2: Synthetic Division.

When the divisor has the form $\boxed{x-a}$ or $\boxed{x+a}$, we can apply the method of synthetic division to obtain the result.

E.g. Divide $2x^3-3x^2-11x+7$ by $x-3$ using synthetic division.

Set up:

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -11 & 7 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & \boxed{1} \end{array}$$

Remainder

constant term

coeff. of x

coeff x^2

Quotient: $2x^2+3x-2$

Remainder: 1

Ex. Use Synthetic Division to divide

$x^3 - 7x - 6$ by $x + 2$.

$$
\begin{array}{r|rrrr}
\boxed{-2} & 1 & 0 & -7 & -6 \\
 & & -2 & 4 & 6 \\
\hline
 & 1 & -2 & -3 & \boxed{0}
\end{array}
\longrightarrow \text{Remainder}
$$

Quotient: $x^2 - 2x - 3$

<u>Observation:</u> $f(x) = x^3 - 7x - 6$

$f(-2) = (-2)^3 - 7 \cdot (-2) - 6$

$= -8 + 14 - 6 = \boxed{0}$

$f(-2) =$ Remainder in this case

In the previous example:

$f(x) = 2x^3 - 3x^2 - 11x + 7$

$f(3) = 2 \cdot (3)^3 - 3 \cdot (3)^2 - 11 \cdot 3 + 7$

$= 54 - 27 - 33 + 7 = \boxed{1} =$ Remainder

$= 54 - 27 - 33 + 7 = \boxed{1}''$