

Obj 3: The Remainder Theorem.

If the polynomial function $f(x)$ is divided by $x-c$, then the remainder is equal to $f(c)$.

Why is this true?

$$f(x) = (x-c) \cdot \text{quotient} + \text{Remainder}.$$

Plug $x = c$ into both sides:

$$f(c) = (c-c) \cdot \text{quotient} + \text{Remainder}$$

$$f(c) = 0 \cdot \text{quotient} + \text{Remainder}$$

$$f(c) = \text{Remainder}$$

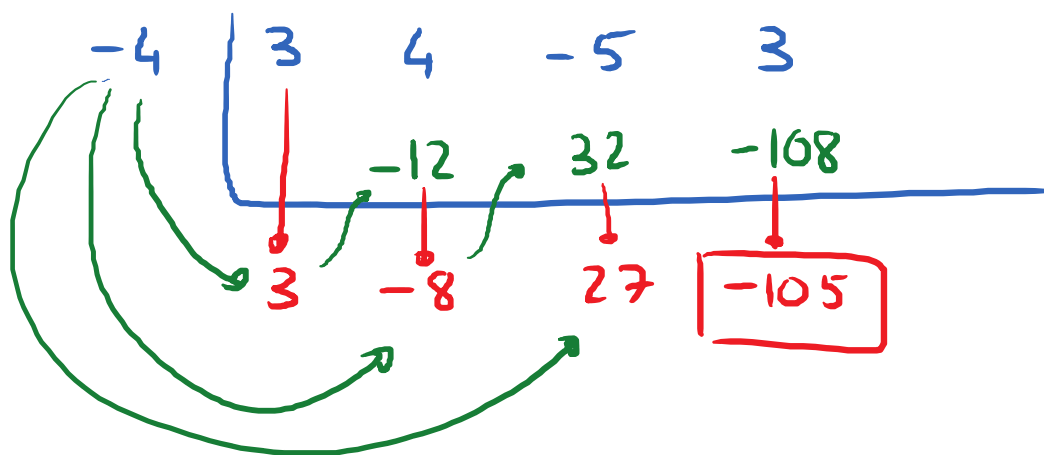
This theorem can help us evaluate polynomial.
 To find $f(c)$, we just need to use synthetic division
 to divide $f(x)$ by $x-c$ and find the remainder.

E.g. Given: $f(x) = 3x^3 + 4x^2 - 5x + 3$.

Use the Remainder Theorem to find $f(-4)$

→ Divide $f(x)$ by $x - (-4)$

→ Divide $f(x)$ by $x + 4$ using synthetic Division
 and find remainder.



The Remainder Theorem says that

$$f(-4) = -105$$

Obj 4: The Factor Theorem

let $f(x)$ be a polynomial.

① If $f(c) = 0$, then $x - c$ is a factor of $f(x)$

② If $x - c$ is a factor of $f(x)$, then $f(c) = 0$

→ $f(c) = 0$ iff $x - c$ is a factor of $f(x)$

In other words,

c is a zero of f if and only if $x - c$ is a factor of $f(x)$

E.g. Solve the equation

$$\underbrace{15x^3 + 14x^2 - 3x - 2 = 0}_{f(x)}$$

Given -1 is a zero of the equation.

How do we find the remaining solutions?

Since $x = -1$ is a zero of $f(x)$, $x+1$ is a factor of $f(x)$. To find the remaining zeros, we just need to find the other factor. To find it, we just need to divide $f(x)$ by $x+1$ using synthetic division.

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & 0 \end{array}$$

The other factor is $15x^2 - x - 2$.

$$\text{Set } 15x^2 - x - 2 = 0$$

$$(3x + 1)(5x - 2) = 0$$

$$\boxed{x = -\frac{1}{3}}; \boxed{x = \frac{2}{5}}$$

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$$x = -\frac{2}{3}; x = -\frac{5}{5}$$