Obj 3: The Remainder Theorem.

If the polynomial function $f(x)$ is divided by $x-c$, then the remainder is equal to $f(c)$.

Why is this true?

$$f(x) = (x-c)\cdot \text{quotient} + \text{Remainder}.$$

Plug $x = c$ into both sides:

$$f(c) = (c-c)\cdot \text{quotient} + \text{Remainder}$$

$$f(c) = 0\cdot \text{quotient} + \text{Remainder}$$

$$f(c) = \text{Remainder}$$

This theorem can help us evaluate polynomial.

To find $f(c)$, we just need to use synthetic division to divide $f(x)$ by $x-c$ and find the remainder.

E.g. Given: $f(x) = 3x^3 + 4x^2 - 5x + 3$.

Use the Remainder Theorem to find $f(-4)$

→ Divide $f(x)$ by $x-(-4)$

→ Divide $f(x)$ by $x+4$ using synthetic Division and find remainder.

$$\begin{array}{r|rrrr} -4 & 3 & 4 & -5 & 3 \\ & & -12 & 32 & -108 \\ \hline & 3 & -8 & 27 & \boxed{-105} \end{array}$$

The Remainder Theorem says that

$$\boxed{f(-4) = -105}$$

Obj 4: The Factor Theorem

Let $f(x)$ be a polynomial.

① If $f(c) = 0$, then $x - c$ is a factor of $f(x)$

② If $x - c$ is a factor of $f(x)$, then $f(c) = 0$

→ $f(c) = 0$ iff $x - c$ is a factor of $f(x)$

In other words,

c is a zero of f if and only if $x - c$ is a factor of $f(x)$

E.g. Solve the equation

$$\underbrace{15x^3+14x^2-3x-2}_{f(x)}=0$$

Given -1 is a zero of the equation. How do we find the remaining solutions?

Since $x=-1$ is a zero of $f(x)$, $x+1$ is a factor of $f(x)$. To find the remaining zeros, we just need to find the other factor. To find it, we just need to divide $f(x)$ by $x+1$ using synthetic division.

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & \boxed{0} \end{array}$$

The other factor is $15x^2-x-2$.

Set $15x^2-x-2=0$

$(3x+1)(5x-2)=0$

$\boxed{x=-\frac{1}{3}}$, $\boxed{x=\frac{2}{5}}$

$\boxed{x = -\frac{\ }{3}}$, $\boxed{x = \frac{\ }{5}}$