3.4-Zeros of Polynomial Functions

Tuesday, November 7, 2017 10:03 AM

Obj 1: Use the Rational Zero Theorem to find possible rational zeros of polynomial functions

Rational Zero Theorem:

$$f(x) = \boxed{a_n} x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + \boxed{a_0}$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$: integers.

If the rational number $\frac{p}{q}$ is a zero of f, then the numerator p must be a factor of the constant term a_0 and the denominator q must be a factor of the leading coefficient a_n

E.g. $f(x) = \boxed{5} x^5 + 7x^4 - 3x^3 + 2x^2 - 8x \boxed{-7}$

Constant term : -7. leading coefficient : 5

Factors of -7 : ± 1; ± 7.

Factors of 5: $\pm 1, \pm 5$.

Hence, possibilities for p: $\pm 1, \pm 7$

possibilities for q: $\pm 1, \pm 5$.

$\rightarrow$ possibilities for $\frac{p}{q}$: $\pm 1, \pm \frac{1}{5}, \pm 7, \pm \frac{7}{5}$.

Rational Zero Theorem says that any rational zero of f must come from the above list of 8 fractions.

E.g. $f(x) = x^3 + x^2 - 5x - 2$.

1. Apply the rational zero theorem to find the rational zero of the function.
2. Find all zeros of f.

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1. Constant Term: -2. leading Coefficient: 1.

Possibilities for p: $\pm 1, \pm 2$

Possibilities for q: ± 1

Possibilities for $\frac{p}{q}$: $\pm 1; \pm 2$

So, any rational zero of f must come from this list of 4 numbers

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & -2 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & \boxed{-5} \end{array}$$

1 is not a zero. (b/c $R = -5 \neq 0$)

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -5 & -2 \\ & & -1 & 0 & 5 \\ \hline & 1 & 0 & -5 & \boxed{3} \end{array}$$

-1 is not a zero (b/c $R = 3 \neq 0$)

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -5 & -2 \\ & & 2 & 6 & 2 \\ \hline & 1 & 3 & 1 & \boxed{0} \end{array}$$

2 is a zero. (b/c $R = 0$)

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -5 & -2 \\ & & -2 & 2 & 6 \\ \hline & 1 & -1 & -3 & \boxed{4} \end{array}$$

-2 is not a zero (b/c $R = 4 \neq 0$)

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -5 & -2 \\ & & -2 & 2 & 6 \\ \hline & 1 & -1 & -3 & \boxed{4} \end{array}$$

-2 is not a zero (b/c $R = 4 \neq 0$)

From this test, we know that $x=2$ is the only rational zero of f.

(2) How do we find the remaining zeros of f?

From the synthetic division with 2, we get:

$$f(x) = (x-2)(x^2+3x+1)$$

$$(x-2)(x^2+3x+1) = 0$$

$x-2=0$ or $\boxed{x^2+3x+1=0}$

$\rightarrow$ this already gave us $x=2$

To find the remaining zeros, we solve: $x^2+3x+1=0$

Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$.

$a=1$; $b=3$, $c=1$.

$$x = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}.$$

Hence, the remaining solutions are:

$$x = \frac{-3-\sqrt{5}}{2}; \quad x = \frac{-3+\sqrt{5}}{2}$$