Ex. $g(x) = x^3 + 4x^2 - 3x - 6$

(1) Use the Rational Zero Theorem to find a rational zero of g. (Given that g only has 1 rational zero.)

(2) Find all zeros of g.

(1) Possibilities for rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -3 & -6 \\ & & 1 & 5 & 2 \\ \hline & 1 & 5 & 2 & \boxed{-4} \end{array}$$

1 is not a zero

$$\begin{array}{r|rrrr} -1 & 1 & 4 & -3 & -6 \\ & & -1 & -3 & 6 \\ \hline & 1 & 3 & -6 & \boxed{0} \end{array}$$

-1 is a zero.

Since we are told that f has only 1 rational zero, we don't need to test any further.

(2) $f(x) = (x+1)(x^2+3x-6)$

To find the other zeros: $x^2+3x-6 = 0$

$$x = \frac{-3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-6)}}{2} = \frac{-3 \pm \sqrt{33}}{2}$$

Zeros of f are: $-1, \frac{-3+\sqrt{33}}{2}, \frac{-3-\sqrt{33}}{2}$.

Obj 2: Properties of Zeros (Roots) of Polynomial Equations

1. If a polynomial function has degree n, it will have n (complex) zeros. (Counting multiplicity)
2. If $a+bi$ is a zero, then $a-bi$ must be a zero as well. The imaginary zeros always occur in conjugate pairs.

E.g. $f(x) = x^4 - x^3 - 2x^2 + 6x - 4$

1. Find all rational zeros of f.
2. Find all zeros of f.

1. Possibilities for rational zeros: $\pm 1, \pm 2, \pm 4$

After testing $x = 1$; $x = -2$ are the only rational zeros of f.

2.
$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -2 & 6 & -4 \\ & & 1 & 0 & -2 & 4 \\ \hline & 1 & 0 & -2 & 4 & \boxed{0} \end{array}$$

$$f(x) = (x-1)(x^3 - 2x + 4)$$

-2 must be a zero for this cubic polynomial

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ & & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

Remaining factor: $x^2 - 2x + 2$

$$f(x) = (x-1)(x+2)(x^2 - 2x + 2)$$

To find remaining zeros: $x^2 - 2x + 2 = 0$

$$x = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$x = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

Zeros of f are: $1, -2, 1+i, 1-i$

Obj 3: linear factorization Theorem.

E.g. Find a third degree polynomial function $f(x)$ with real coefficients such that -3 and i are zeros of f and $\boxed{f(1) = -24}$

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* Since -3 is a zero of f, $x+3$ must be a factor.
* Since i is a zero of f, $x-i$ must be a factor
* Since imaginary zeros always come in pairs, $-i$ must be a factor of f. Since $-i$ is a zero of f, $x+i$ must be a factor

So, $f(x) = a(x+3)(x-i)(x+i)$

$$= a(x+3)\left[x^2 - i^2\right]$$

$$= a(x+3)(x^2-(-1)) = a(x+3)(x^2+1)$$

$$f(x) = a(x^3+3x^2+x+3)$$

$$f(1) = a \cdot (1 + 3 + 1 + 3)$$

$$f(1) = 8a = -24$$

$$\boxed{a = -3}$$