Recall: Things to do for Rational Functions.

* Find domain: Set bottom $= 0$.

  Solve for x.

  Exclude those values of x from domain.

* Find vertical asymptotes:

  • Factor top and bottom completely.

  • Cancel.

  • Set bottom of canceled expression $= 0$

E.x. $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 9}$.

Find all the vertical asymptotes of f.

$$f(x) = \frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{x-2}{x+3}$$

Set $x + 3 = 0$. $x = -3$.

V.A. : $\boxed{x = -3}$

* Find Horizontal Asymptotes:

- $\cdot$ Deg top $<$ Deg bottom. H.A. $y = 0$

- $\cdot$ Deg top $>$ Deg bottom. No H.A.

- $\cdot$ Deg top $=$ Deg bottom. H.A. $y = \dfrac{\text{leading coeff. top}}{\text{leading coeff. bottom}}$

E.x. Find all the H.A.(s) of the given function

① $f(x) = \dfrac{-2x + 1}{3x + 5}$ ③ $h(x) = \dfrac{12x}{3x^2 + 1}$

② $g(x) = \dfrac{9x^3}{3x^2 + 1}$

① $f(x)$.   H.A. $y = \dfrac{-2}{3} = -\dfrac{2}{3}$.

② $g(x)$.   H.A. None

③ $h(x)$.   H.A. $y = 0$.

$\longrightarrow$ Put everything together to graph rational functions

Steps for Graphing Rational Functions.

① Find Domain. (Set bottom = 0. Exclude those x)

② Find x-intercepts and the y-intercept.

• To find x-intercept(s): Set $y = 0$. This implies that top = 0. Then we solve for x.

• To find y-intercept: Set $x = 0$.

③ Find the vertical asymptote(s) if any.

④ Find the horizontal asymptote(s) if any.

⑤ Plot points between and beyond each x-intercept and vertical asymptote(s).

⑥ Graph the function

E.g. Go through all the steps described above and graph the function: $f(x) = \dfrac{3x - 3}{x - 2}$.

(1) Domain   Set $x - 2 = 0$. $x = 2$.

Domain in interval notation: $(-\infty, 2) \cup (2, \infty)$

(2) x-intercept(s) and y-intercept.

x-intercept: Set $3x - 3 = 0$. $x = 1$.

x-intercept: $\boxed{(1, 0)}$

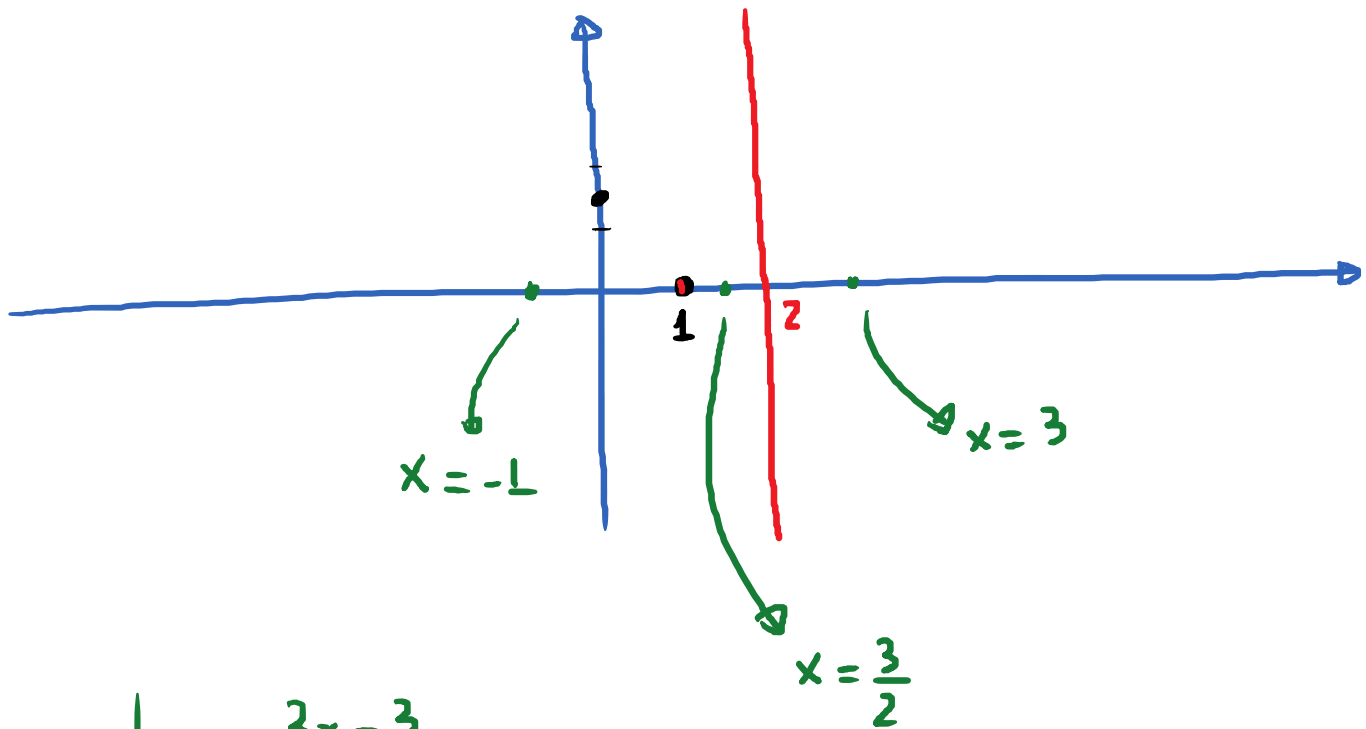y-intercept: Set $x = 0$ in $f(x)$: $\dfrac{3 \cdot (0) - 3}{0 - 2} = \dfrac{3}{2}$

y-intercept: $\boxed{\left(0, \dfrac{3}{2}\right)}$

(3) V.A. $f(x) = \dfrac{3(x - 1)}{x - 2}$.   Set $x - 2 = 0$. $x = 2$

V.A. $\boxed{x = 2}$

(4) H.A. $\boxed{y = 3}$ $\left(\deg \text{top} = \deg \text{bottom} = 1\right)$

(5) Choose points

$x = -1$

$x = 3$

$x = \frac{3}{2}$

x	$y = \dfrac{3x-3}{x-2}$
-1	2
$\dfrac{3}{2}$	-3
3	6

⑥ Graph the function

$$f(x) = \frac{3x-3}{x-2}$$
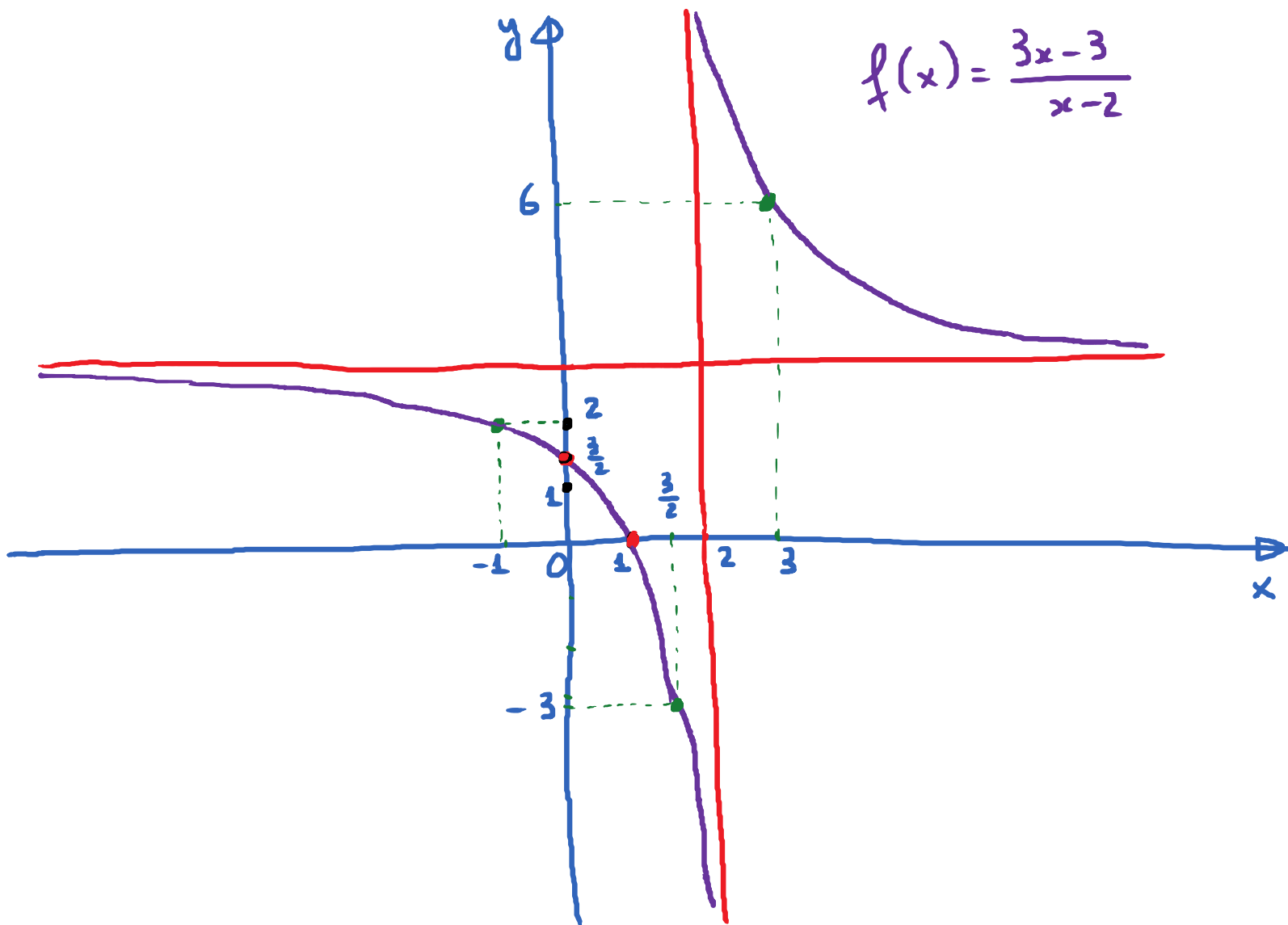
E. x. Go through these steps to graph the given function.

① $f(x) = \dfrac{-x}{x+1}$.   ② $g(x) = -\dfrac{1}{x^2-4}$