3.5-Rational Functions and their graphs

Thursday, November 9, 2017 9:59 AM

Obj 1: Rational Functions and Domains of Rational Functions.

A rational function is a function of the form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials.

E.g. $$f(x) = \frac{5x^2 - 12x + 6}{x^3 + 4x^2 - 4x + 8}$$

$$g(x) = \frac{5}{x} \quad ; \quad h(x) = \frac{x+1}{2x^2 + 7}$$

Find domains of Rational Function.

$$f(x) = \frac{p(x)}{q(x)}$$

To find domain: Step 1: Set denominator $q(x) = 0$

Solve for x.

Step 2: Domain = all real numbers except for the values of x in Step 1.

<u>E.g.</u> Find the domain of the given function

(a) $f(x) = \dfrac{x-5}{x^2-25}$

(b) $g(x) = \dfrac{x^2-25}{x-5}$

(c) $h(x) = \dfrac{x+7}{x^2+49}$

(d) $u(x) = \dfrac{x^2+x+4}{x^3-x^2+3x-3}$

<u>Sol:</u> (a) $x^2-25=0$; $x^2=25$; $x=\pm 5$

<u>Domain:</u> All real numbers except 5 and -5

Interval notation: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

(b) $x-5=0$; $x=5$.

<u>Domain:</u> $(-\infty,5)\cup(5,\infty)$

(c) $x^2+49=0$; $x^2=-49$; $x=\underbrace{\pm 7i}_{\text{imaginary}}$.

<u>Domain:</u> All real numbers.

Interval Notation: $(-\infty,\infty)$

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(d) $\underbrace{x^3 - x^2}+\underbrace{3x-3} = 0$

$x^2(x-1) + 3(x-1) = 0$

$(x-1)(x^2+3) = 0$

$x - 1 = 0$ or $x^2 + 3 = 0$

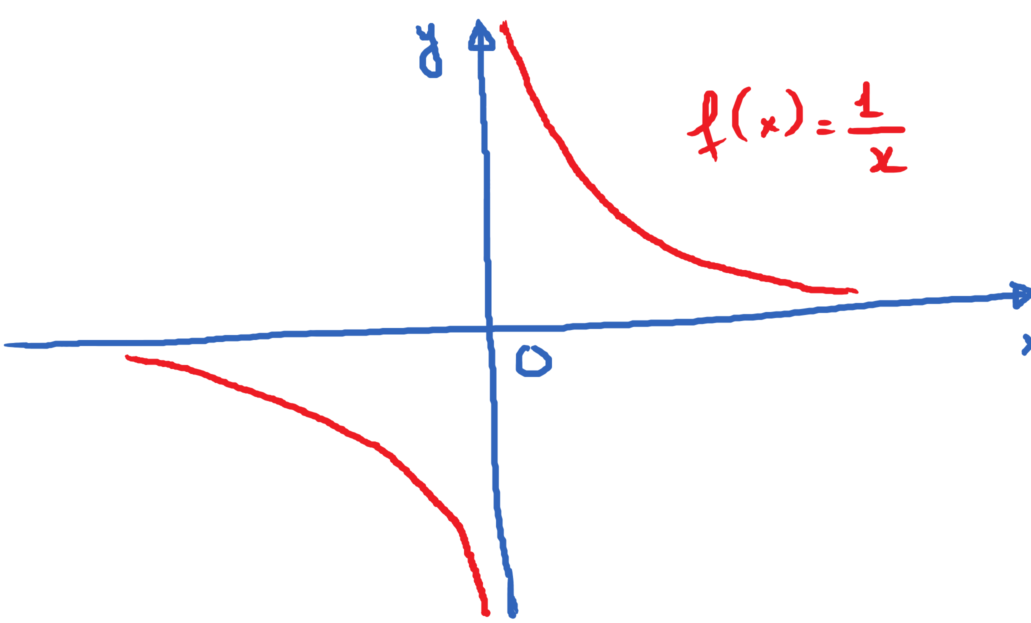
$x = 1$ $\qquad$ $x^2 = -3$

$x = \pm i\sqrt{3} \rightarrow$ imaginary

Conclusion: Domain $= (-\infty, 1) \cup (1, \infty)$

Obj 2: Vertical Asymptotes and Horizontal Asymptotes.

E.g. $f(x) = \frac{1}{x}$



x	$f(x) = \frac{1}{x}$
1	1
0.1	10
0.01	100
0.001	1000
0.0001	10000

As x gets closer and closer to zero from the right side, the value of the function gets larger and larger.

Arrow notation:

As $x \longrightarrow 0^+$, $f(x) \longrightarrow \infty$

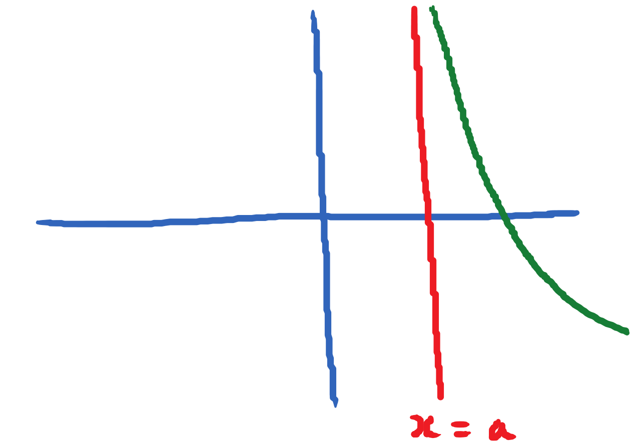
Similarly,

As $x \longrightarrow 0^-$, $f(x) \longrightarrow -\infty$

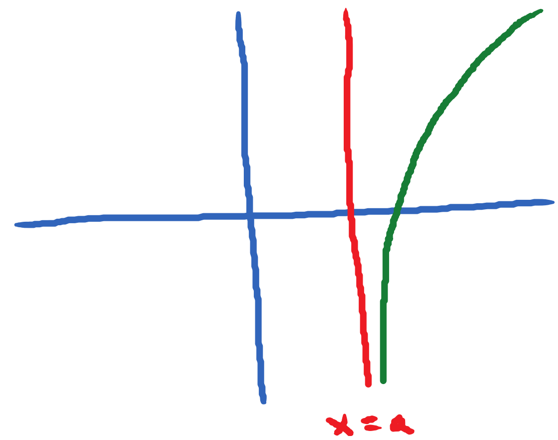
(as x gets closer and closer to 0 from the left, $f(x)$ gets larger and larger negatively)

Definition of a vertical asymptote:

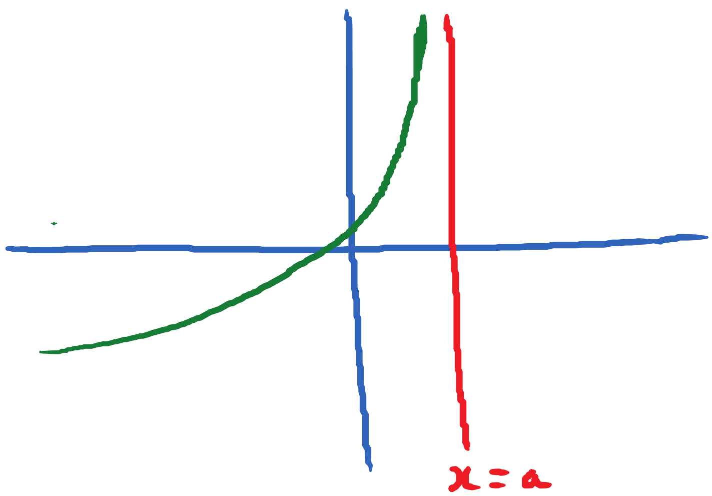
We say that the vertical line $x = a$ is a vertical asymptote of the function $y = f(x)$ if one of the followings happens.



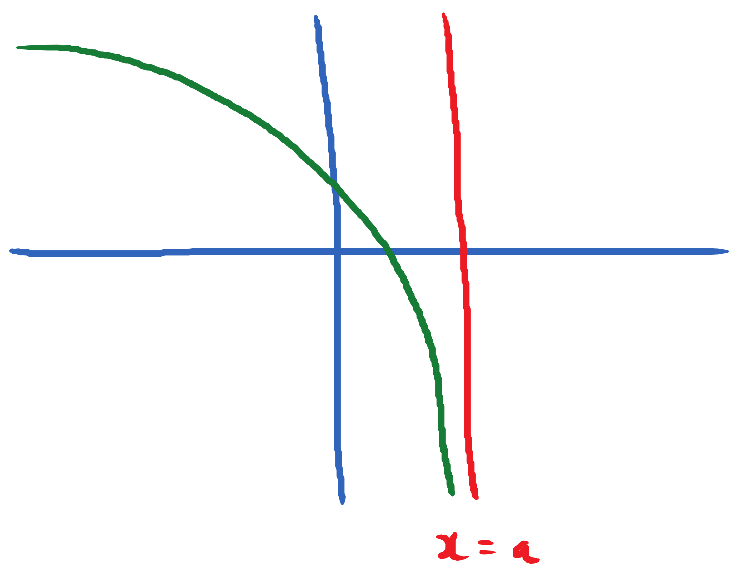
As $x \to a^+$, $f(x) \to \infty$



As $x \to a^+$, $f(x) \to -\infty$



As $x \to a^-$, $f(x) \to \infty$



As $x \to a^-$, $f(x) \to -\infty$

Q: How do we find vertical asymptotes?

Process: Step 1: Factor numerator and denominator completely.

Step 2: Cancel common factor(s) between top and bottom.