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Step 3: Set denominator of the canceled expression equal 0.

E.g. Find the V.A.(s) of the given function.

ⓐ $f(x) = \dfrac{x}{x^2-1}$ ⓑ $g(x) = \dfrac{x+1}{x^2-1}$

ⓒ $h(x) = \dfrac{x^2+3x}{x^2+4x}$ ⓓ $u(x) = \dfrac{x}{x^2+4}$

Sol: ⓐ $f(x) = \dfrac{x}{(x+1)(x-1)}$ ← Factor

Set $(x+1)(x-1)=0$; $x+1=0$ or $x-1=0$

V.A. : $\boxed{x=-1}$ $\boxed{x=1}$

ⓑ $g(x) = \dfrac{\cancel{x+1}}{\cancel{(x+1)}(x-1)} = \dfrac{1}{x-1}$.

Set $x-1=0$. $\boxed{x=1}$ V.A.

ⓒ $h(x) = \dfrac{\cancel{x}(x+3)}{\cancel{x}(x+4)} = \dfrac{x+3}{x+4}$.

Set $x+4=0$; V.A. $\boxed{x=-4}$

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(d) $u(x) = \dfrac{x}{x^2+4}$ → cannot be factor over the real

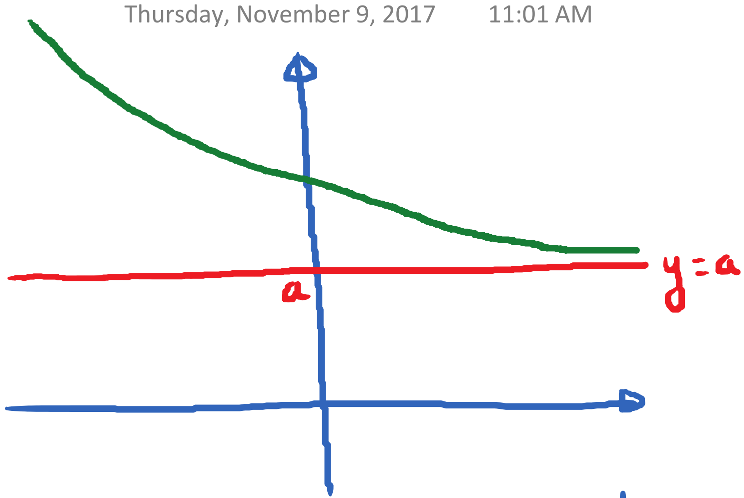
Set $x^2+4=0$; $x^2=-4$; $x=\underbrace{\pm 2i}_{\text{imaginary}}$

No Vertical Asymptote.

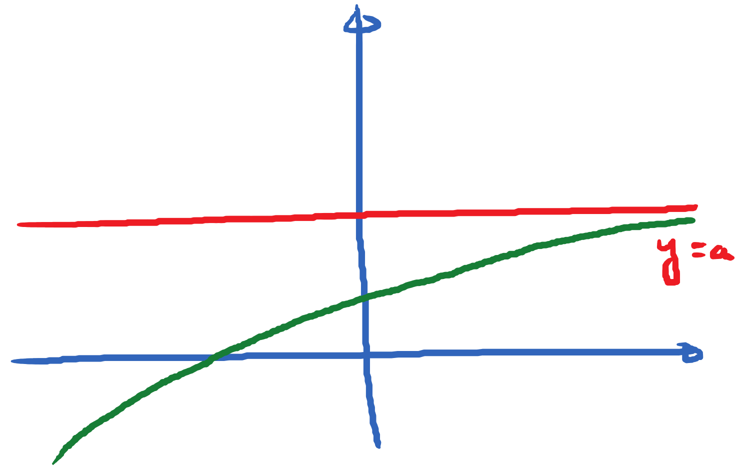
Horizontal Asymptotes.

<u>Definition of a horizontal asymptote:</u>

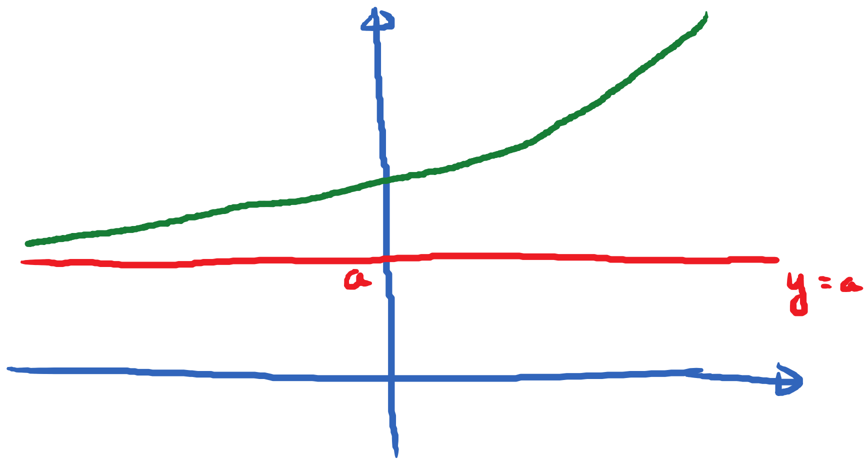
The horizontal line $y=a$ is a horizontal asymptote of $y=f(x)$ if one of the followings happens



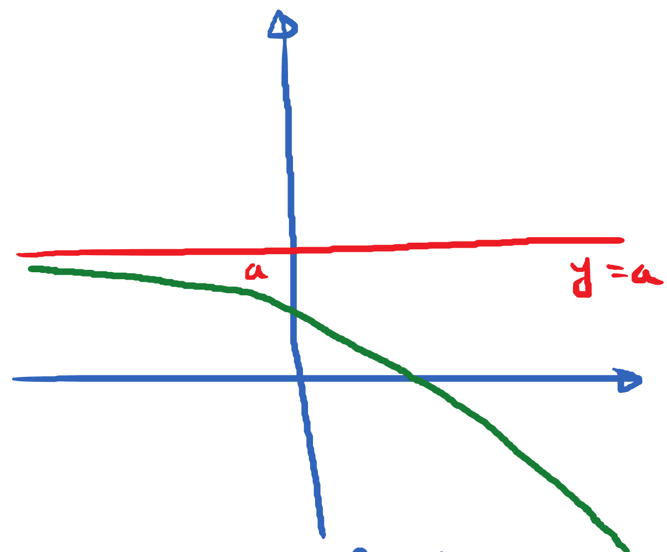
As $x \to \infty$, $f(x) \to a^+$



As $x \to \infty$, $f(x) \to a^-$



As $x \to -\infty$, $f(x) \to a^+$



As $x \to -\infty$, $f(x) \to a^-$

How to find horizontal asymptotes.

Scenario 1:

* $f(x) = \dfrac{9x^2 + x + 1}{3x^2 + 10} \approx \dfrac{9\cancel{x^2}}{3\cancel{x^2}} = 3$

Find H.A. of f:

As x gets large, $f(x)$ gets close to 3.

Hence, the horizontal asymptote is $\boxed{y = 3}$

* $g(x) = \dfrac{2x^4 + x^2 + 5}{7x^4 + 3x^3 - 1} \approx \dfrac{2\cancel{x^4}}{7\cancel{x^4}} = \dfrac{2}{7}$.

H.A. $\boxed{y = \dfrac{2}{7}}$

If degree top = degree bottom, then the H.A. is

$$y = \frac{\text{leading coeff. top}}{\text{leading coeff. bottom}}$$

Scenario 2: $f(x) = \dfrac{x^2 + x + 1}{4x^4 + x^3 - 5} \approx \dfrac{x^2}{4x^4} = \dfrac{1}{4x^2}$

$\longrightarrow 0$ when x is large.

Hence, H.A. $\boxed{y = 0}$

If deg top < deg bottom, then the H.A. is $y = 0$.

Scenario 3: $f(x) = \dfrac{x^4 + 2}{x^2 + 5} \approx \dfrac{x^4}{x^2} = x^2$

gets larger and larger indefinitely

Conclusion: No H.A.

If deg top > deg bottom, then there is NO H.A.