1

Trigonometric Functions

Trigonometry

LIAL | HORNSBY | SCHNEIDER | DANIELS

ELEVENTH EDITION

PEARSON
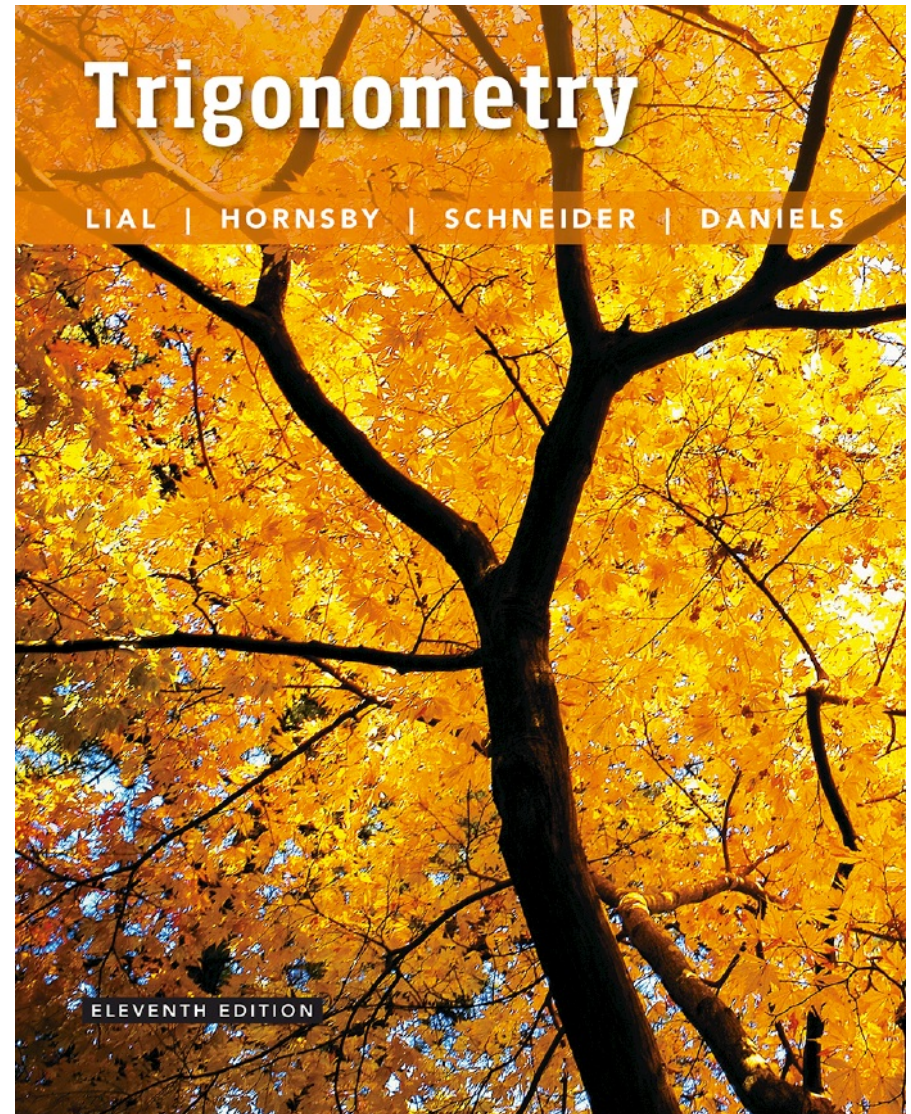
1.1 Angles

Basic Terminology ▪ Degree Measure ▪ Standard Position ▪ Coterminal Angles

Basic Terminology

Two distinct points determine **line *AB.***

$$A \qquad\qquad B$$

Line segment *AB* is a portion of the line between *A* and *B*, including points *A* and *B.*

$$A \qquad\qquad B$$

Ray *AB* is a portion of line *AB* that starts at *A* and continues through *B*, and on past *B.*
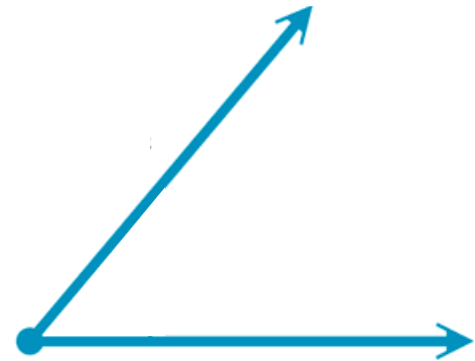

$$A \qquad\qquad B$$

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Basic Terminology

An **angle** consists of two rays in a plane with a common endpoint.

The two rays are the **sides** of the angle.

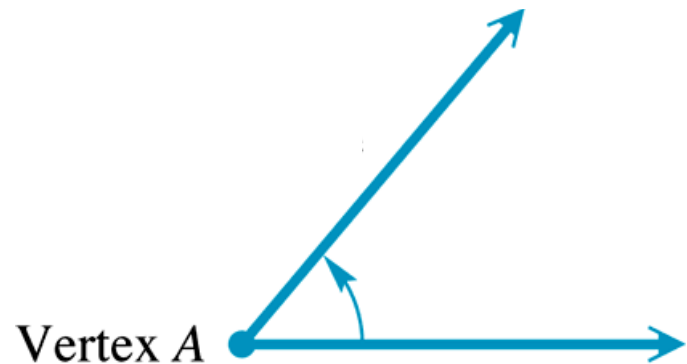The common endpoint is called the **vertex** of the angle.

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Basic Terminology

An angle's **measure** is generated by a rotation about the vertex.

The ray in its initial position is called the **initial side** of the angle.

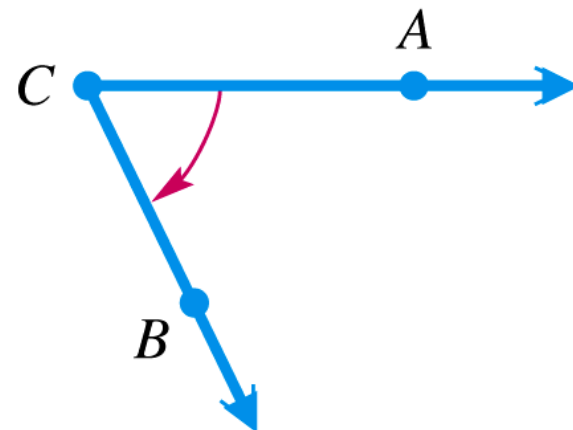The ray in its location after the rotation is the **terminal side** of the angle.

Vertex *A*

Basic Terminology

Positive angle: The rotation of the terminal side of an angle is counterclockwise.

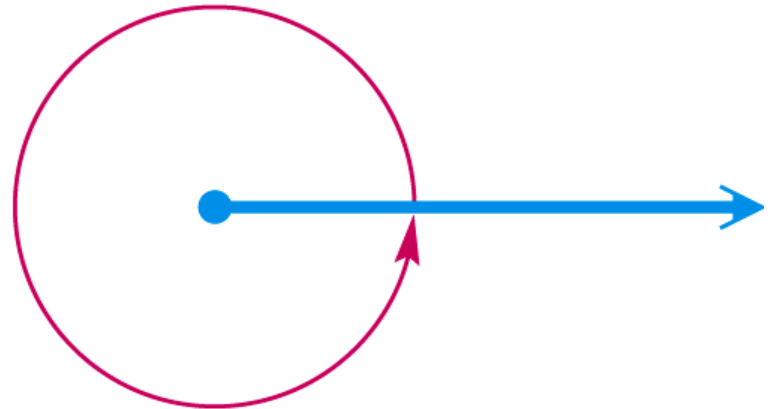**Negative angle**: The rotation of the terminal side is clockwise.
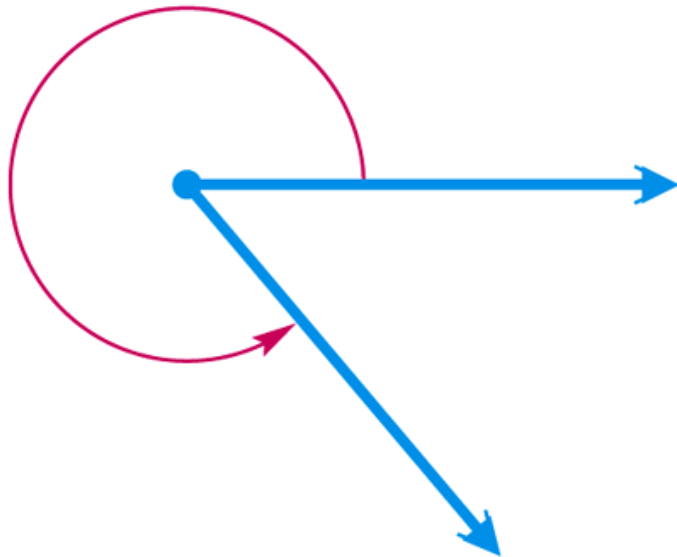
Positive angle

Negative angle

Degree Measure

The most common unit for measuring angles is the **degree**.
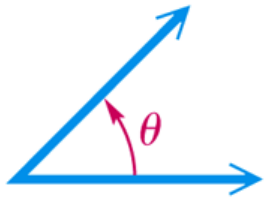
A complete rotation of a
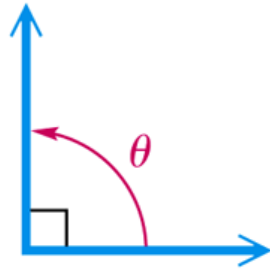ray gives an angle
whose measure is 360°.

$\dfrac{1}{360}$ of complete rotation gives an angle
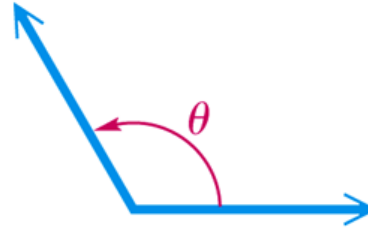whose measure is 1°.

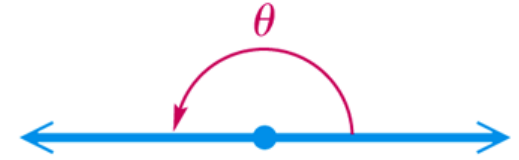# Degree Measure

Angles are classified by their measures.

Acute angle
$0° < \theta < 90°$

Right angle
$\theta = 90°$

Obtuse angle
$90° < \theta < 180°$

Straight angle
$\theta = 180°$

For an angle measuring 40°, find the measure of **(a)** its complement and **(b)** its supplement.

(a) To find the measure of its complement, subtract the measure of the angle from 90°.

$$90° - 40° = 50°$$ Complement of 40°

(b) To find the measure of its supplement, subtract the measure of the angle from 180°.

$$180° - 40° = 140°$$ Supplement of 40°

Find the measure of each marked angle.

Since the two angles form a right angle, they are complementary.

$(6x)°$

$(3x)°$

(a)
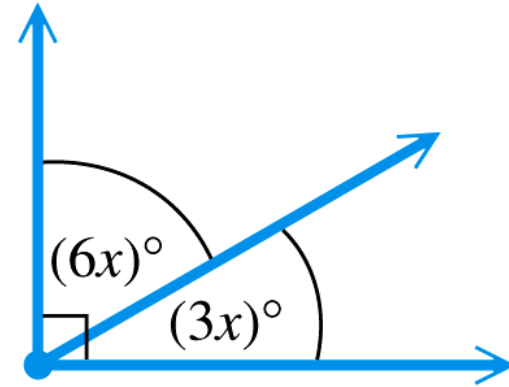
$$6x + 3x = 90$$

$$9x = 90 \qquad \text{Combine like terms.}$$

$$x = 10 \qquad \text{Divide by 9.}$$

Determine the measure of each angle by substituting 10 for x: $6(10) = 60°$ $3(10) = 30°$

Find the measure of each marked angle.

Since the two angles form a straight angle, they are supplementary.

$(4x)°$ $(6x)°$

(b)
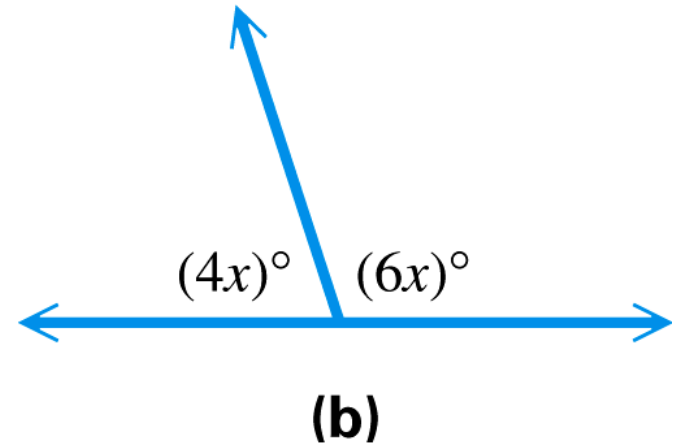
$$4x + 6x = 180$$
$$10x = 180$$
$$x = 18$$

The angle measures are 4(18) = 72 degrees and 6(18) = 108 degrees.

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Degrees, Minutes, Seconds

One **minute** is 1/60 of a degree.

$$1' = \frac{1}{60}^{\circ} \quad \text{or} \quad 60' = 1^{\circ}$$

One **second** is 1/60 of a minute.

$$1'' = \frac{1}{60}' = \frac{1}{3600}^{\circ} \quad \text{or} \quad 60'' = 1'$$

Perform each calculation.

(a) $51°29' + 32°46'$

$$\begin{array}{r} 51°29' \\ + 32°46' \\ \hline 83°75' \end{array}$$

Add degrees and minutes separately.

$$83°75' = 83° + 1°15'$$
$$= 84°15'$$

(b) $90° - 73°12'$

$$\begin{array}{r} 89°60' \\ - 73°12' \\ \hline 16°48' \end{array}$$

Write 90° as 89°60'.

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(a) Convert 74°08′14″ to decimal degrees to the nearest thousandth.

$$74°08′14″ = 74° + \frac{8}{60}^{\circ} + \frac{14}{3600}^{\circ}$$

$$\approx 74° + 0.1333° + 0.0039°$$

$$\approx 74.137°$$

(b) Convert 34.817° to degrees, minutes, and seconds.

$$34.817° = 34° + 0.817°$$

$$= 34° + 0.817(60′)$$

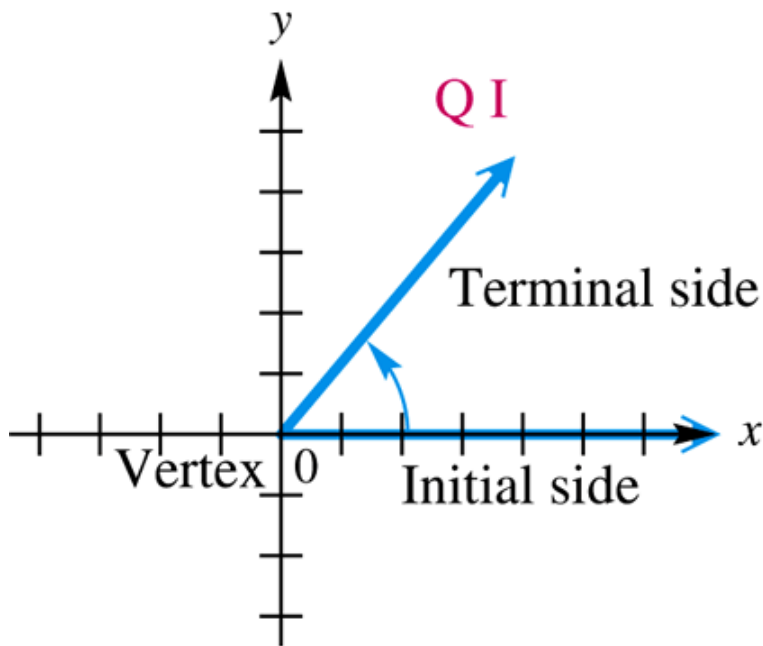$$= 34° + 49.02′$$

$$= 34° + 49′ + 0.02′$$

$$= 34° + 49′ + 0.02(60″)$$

$$= 34° + 49′ + 1.2″$$
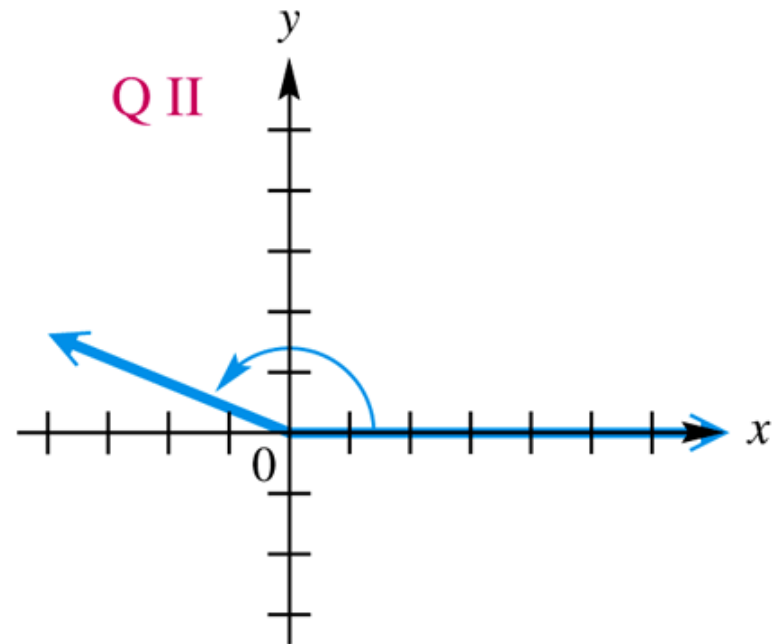
$$\approx 34° 49′ 01″$$

Approximate to the nearest second.

Standard Position

An angle is in **standard position** if its vertex is at the origin and its initial side lies along the positive *x*-axis.


(a) (b)

Quandrantal Angles

Angles in standard position whose terminal sides lie along the *x*-axis or *y*-axis, such as angles with measures 90°, 180°, 270°, and so on, are called **quadrantal angles.**

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Coterminal Angles

A complete rotation of a ray results in an angle measuring 360°. By continuing the rotation, angles of measure larger than 360° can be produced. Such angles are called **coterminal angles.**
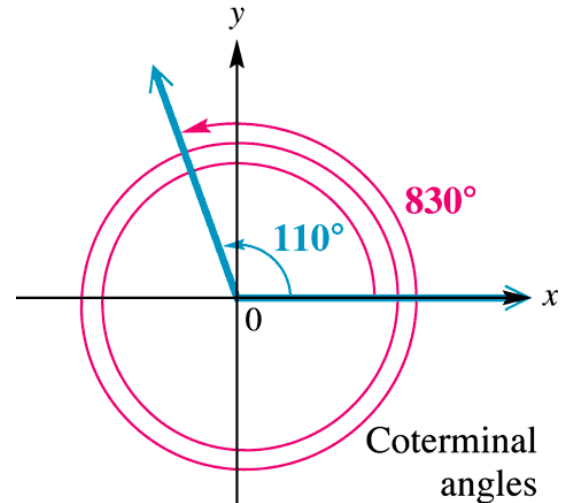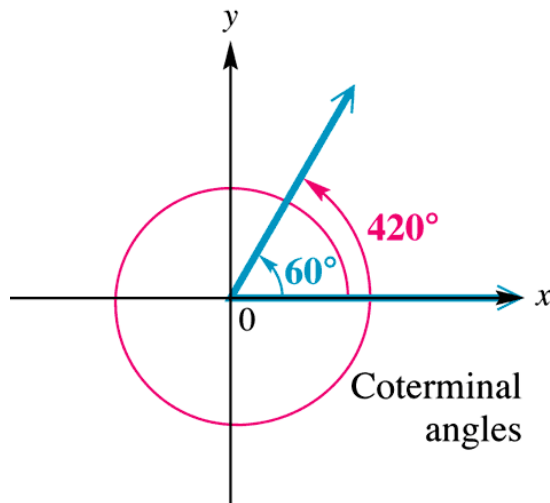
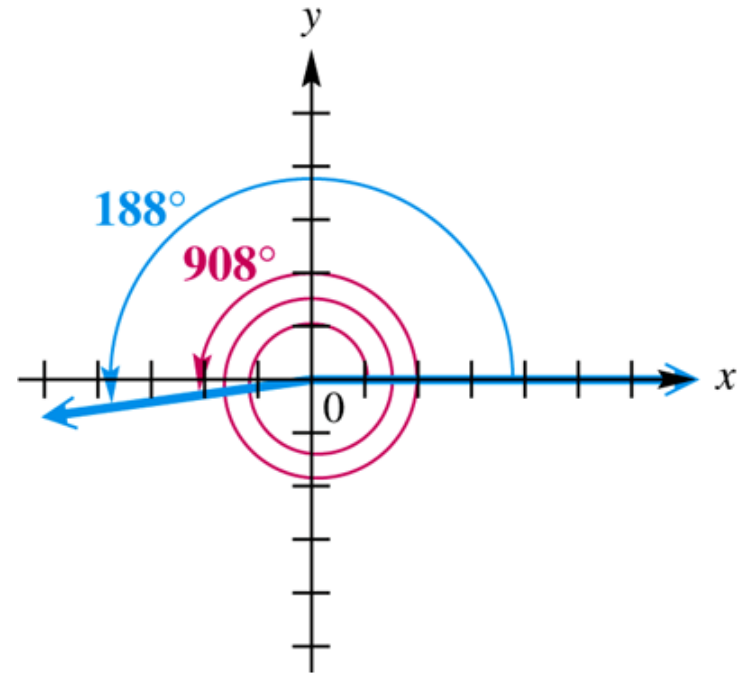The measures of coterminal angles differ by a multiple of 360°.

(a) Find the angle of least positive measure coterminal with an angle of 908°.

Subtract 360° as many times as needed to obtain an angle with measure greater than 0° but less than 360°.

$$908° - 2 \cdot 360° = 188°$$

An angle of 188° is coterminal with an angle of 908°.

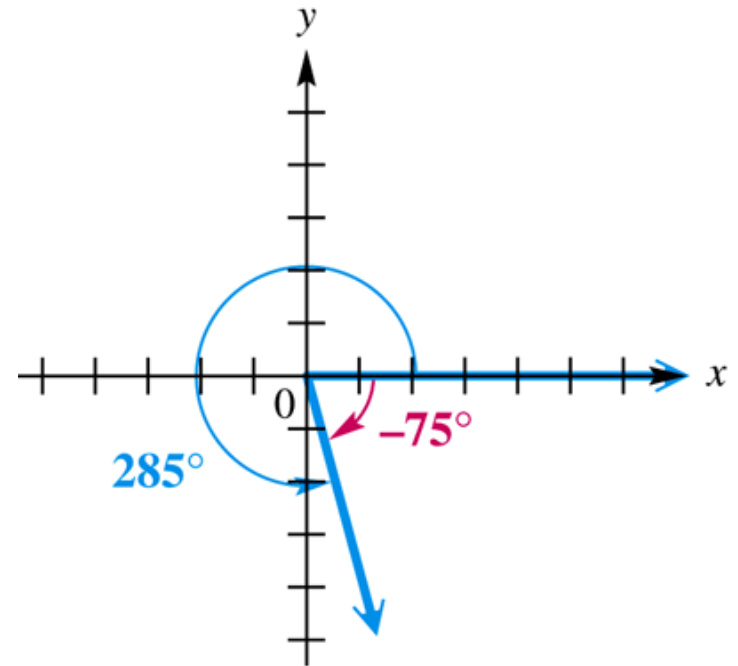**(b)** Find the angle of least positive measure coterminal with an angle of –75°.

$$360° + (-75°) = 285°$$

An angle of –75 °is coterminal with an angle of 285°.

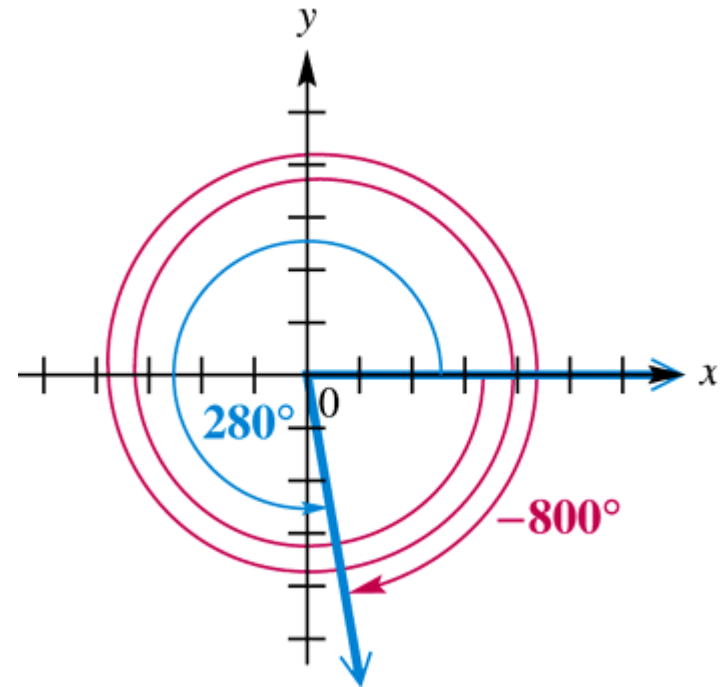**(c)** Find the angle of least positive measure coterminal with an angle of –800°.

The least integer multiple of 360° greater than 800° is

$$360° \cdot 3 = 1080°.$$

$$1080° + (-800°) = 280°$$

An angle of –800° is coterminal with an angle of 280°.

Coterminal Angles

To find an expression that will generate all angles coterminal with a given angle, add integer multiples of 360° to the given angle.

For example, the expression for all angles coterminal with 60° is $60° + n \cdot 360°$.

Coterminal Angles

Value of n	Angle Coterminal with $60°$
2	$60° + 2 \cdot 360° = 780°$
1	$60° + 1 \cdot 360° = 420°$
0	$60° + 0 \cdot 360° = 60°$ (the angle itself)
-1	$60° + (-1) \cdot 360° = -300°$

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A constant angular velocity disk drive spins a disk at a constant speed. Suppose a disk makes 480 revolutions per min. Through how many degrees will a point on the edge of a disk move in 2 sec?

The disk revolves 480 times in 1 min or $\frac{480}{60}$ times = 8 times per sec.

In 2 sec, the disk will revolve 2(8) = 16 times.

Each revolution is 360°, so a point on the edge of the disk will revolve $16 \cdot 360° = 5760°$ in 2 sec.