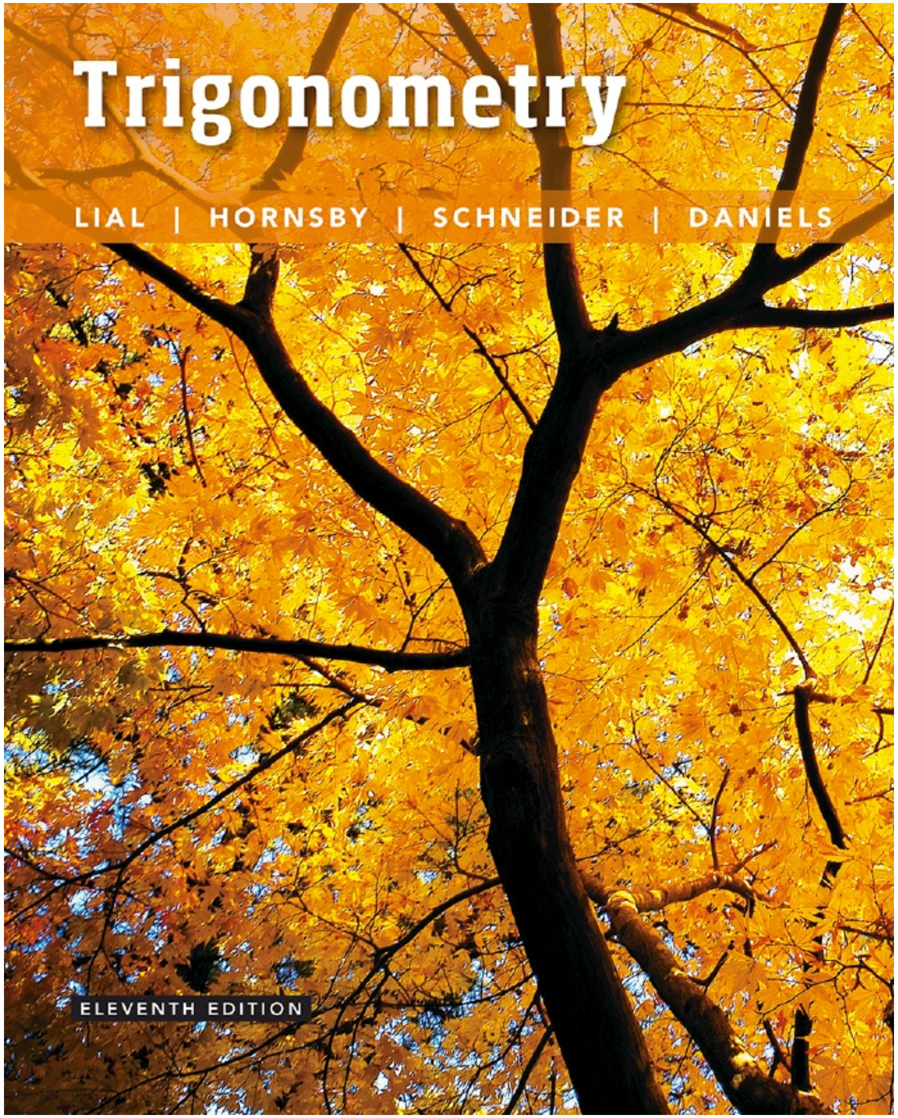
1

Trigonometric Functions



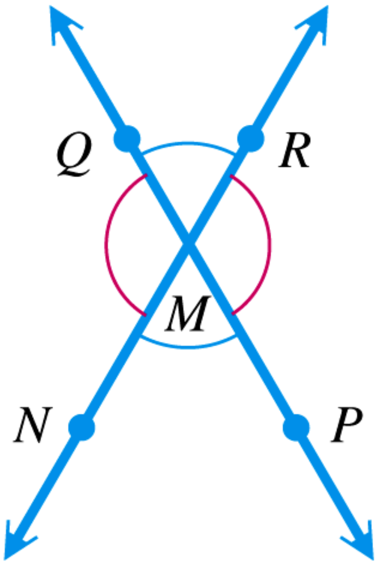
1.2 Angle Relationships and Similar Triangles

Geometric Properties ▪ Triangles

Vertical Angles

Vertical angles have equal measures.

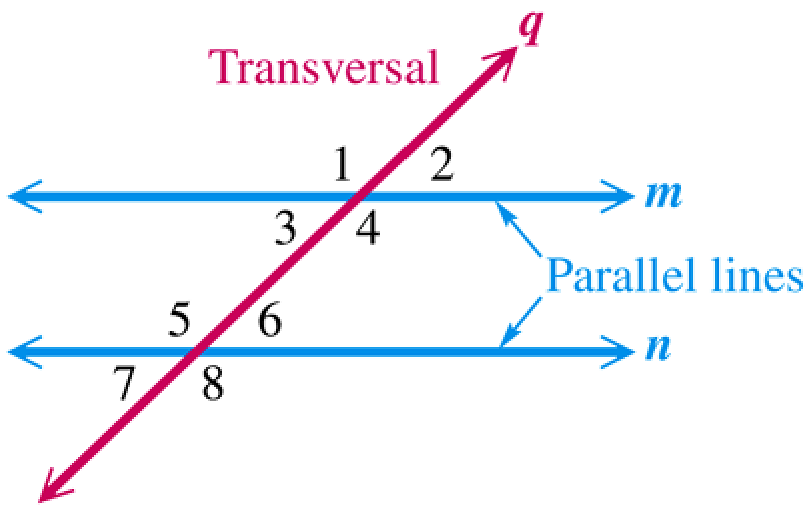
The pair of angles *NMP* and *RMQ* are vertical angles.



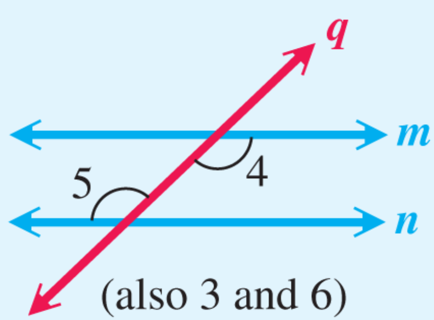
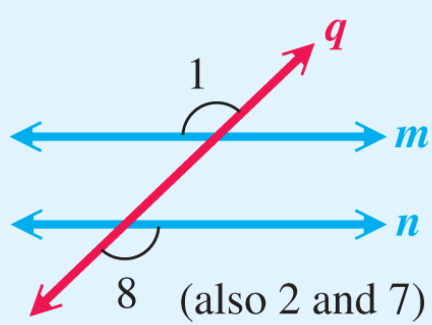
Parallel Lines

Parallel lines are lines that lie in the same plane and do not intersect.

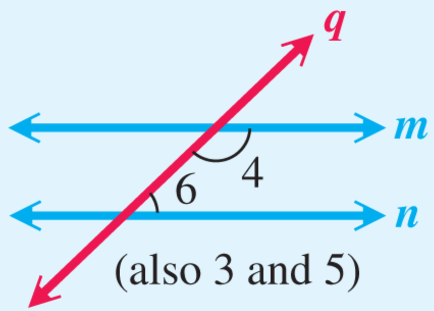
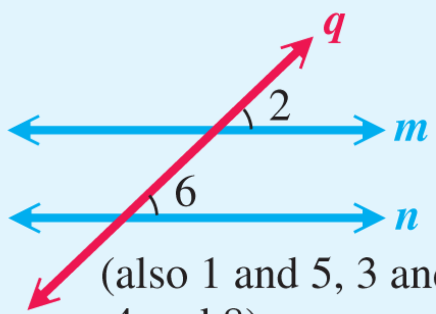
When a line *q* intersects two parallel lines, *q* is called a **transversal.**



Angles and Relationships

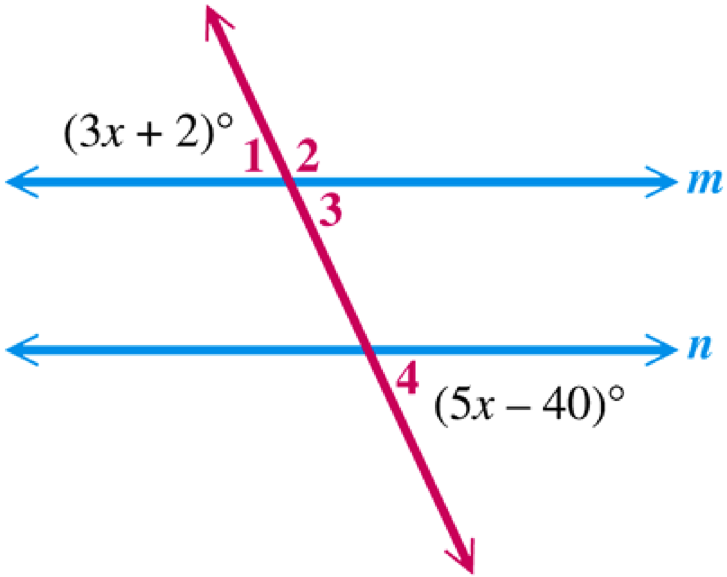
Name	Sketch	Rule
Alternate interior angles	 (also 3 and 6)	Angle measures are equal.
Alternate exterior angles	 (also 2 and 7)	Angle measures are equal.

Angles and Relationships

Name	Sketch	Rule
Interior angles on same side of transversal	 (also 3 and 5)	Angle measures add to 180°.
Corresponding angles	 (also 1 and 5, 3 and 7, 4 and 8)	Angle measures are equal.

Example 1 FINDING ANGLE MEASURES

Find the measures of angles 1, 2, 3, and 4, given that lines *m* and *n* are parallel.



Angles 1 and 4 are alternate exterior angles, so they are equal.

$$3x + 2 = 5x - 40$$

$$42 = 2x$$ Subtract 3*x*. Add 40.

$$21 = x$$ Divide by 2.

Angle 1 has measure

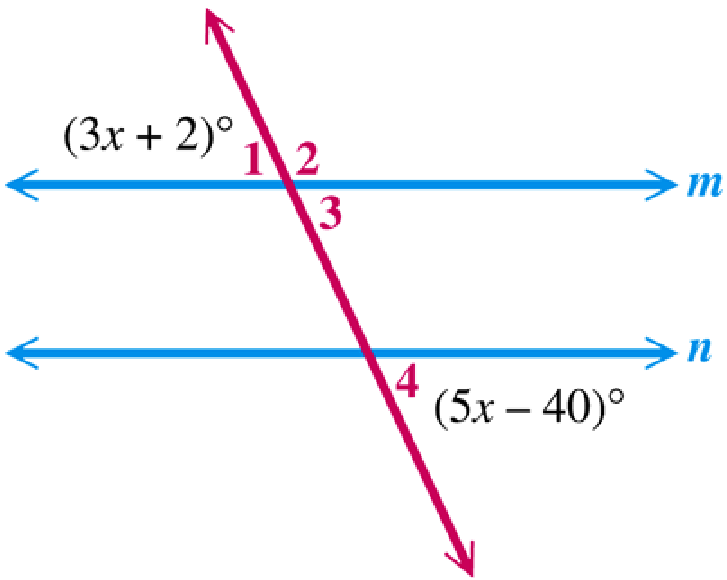
$$3x + 2 = 3(21) + 2 = 65^\circ$$ Substitute 21 for *x*.

Example 1 FINDING ANGLE MEASURES (continued)

Angle 4 has measure

$$5x - 40 = 5(21) - 40 = 65^\circ$$

Substitute 21 for *x*.



Angle 2 is the supplement of a 65° angle, so it has measure $180^\circ - 65^\circ = 115^\circ$.

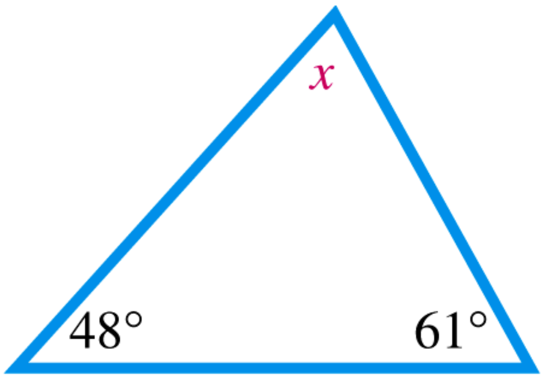
Angle 3 is a vertical angle to angle 1, so its measure is 65°.

Angle Sum of a Triangle

The sum of the measures of the angles of any triangle is 180°.

APPLYING THE ANGLE SUM OF A TRIANGLE PROPERTY

The measures of two of the angles of a triangle are 48° and 61°. Find the measure of the third angle, *x*.



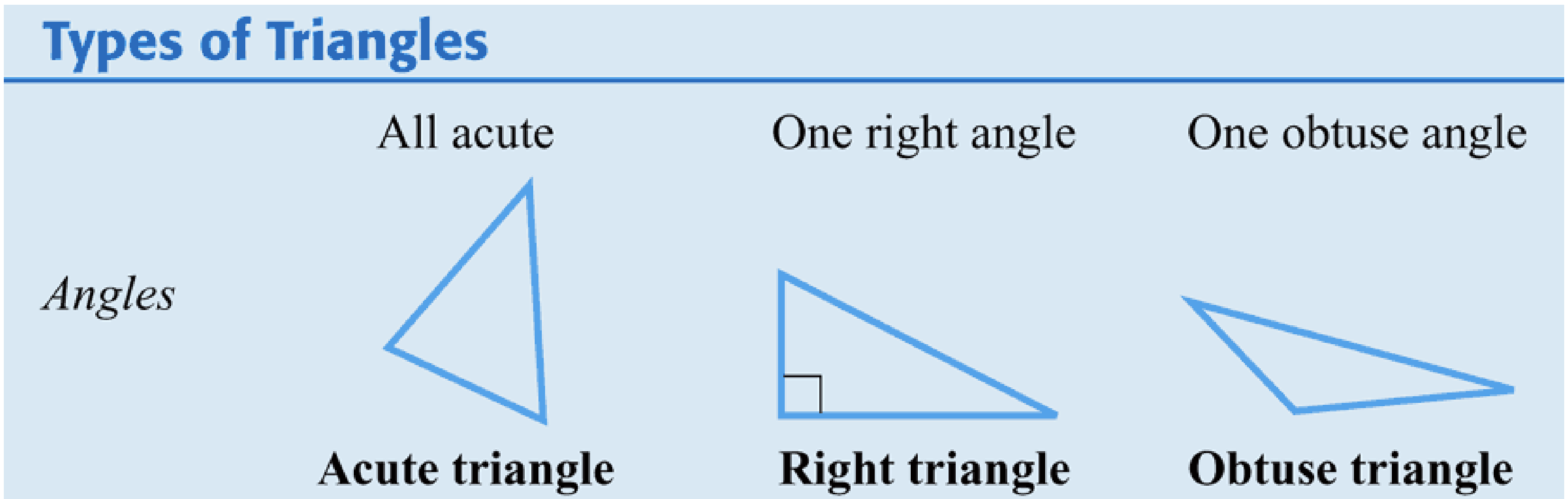
$$48° + 61° + x = 180°$$ The sum of the angles is 180°.

$$109° + x = 180°$$ Add.

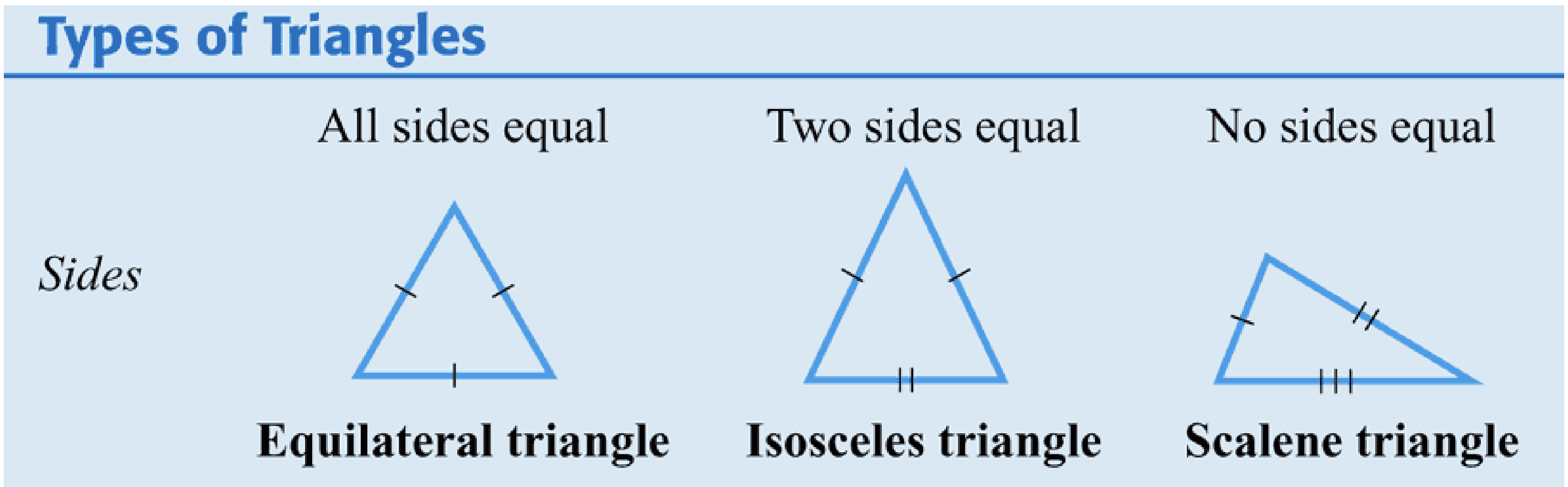
$$x = 71°$$ Subtract 109°.

The third angle of the triangle measures 71°.

Types of Triangles: Angles



Types of Triangles: Sides



Conditions for Similar Triangles

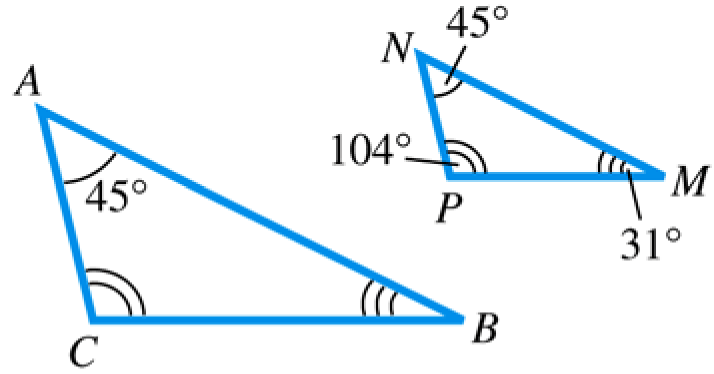
For triangle *ABC* to be similar to triangle *DEF*, the following conditions must hold.

1. Corresponding angles must have the same measure.

2. Corresponding sides must be proportional. (That is, the ratios of their corresponding sides must be equal.)

▶ Example 3 FINDING ANGLE MEASURES IN SIMILAR TRIANGLES

In the figure, triangles *ABC* and *NMP* are similar. Find the measures of angles *B* and *C*.



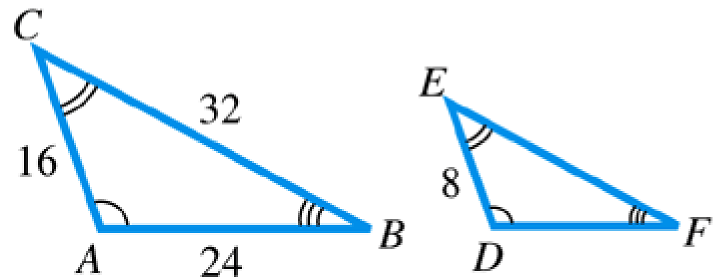
Since the triangles are similar, corresponding angles have the same measure.

C corresponds to *P*, so angle *C* measures 104°.

B corresponds to *M*, so angle *B* measures 31°.

Example 4 FINDING SIDE LENGTHS IN SIMILAR TRIANGLES

Given that triangle *ABC* and triangle *DFE* are similar, find the lengths of the unknown sides of triangle *DFE.*



Similar triangles have corresponding sides in proportion.

DF corresponds to *AB*, and *DE* corresponds to *AC*, so

$$\frac{DE}{AC} = \frac{DF}{AB} \Rightarrow \frac{8}{16} = \frac{DF}{24}$$

Example 4 FINDING SIDE LENGTHS IN SIMILAR TRIANGLES (continued)

$$\frac{8}{16}=\frac{DF}{24} \Rightarrow 8\cdot 24 = 16\cdot DF \Rightarrow 192 = 16\cdot DF \Rightarrow 12 = DF$$

Side *DF* has length 12.

EF corresponds to *CB*, so

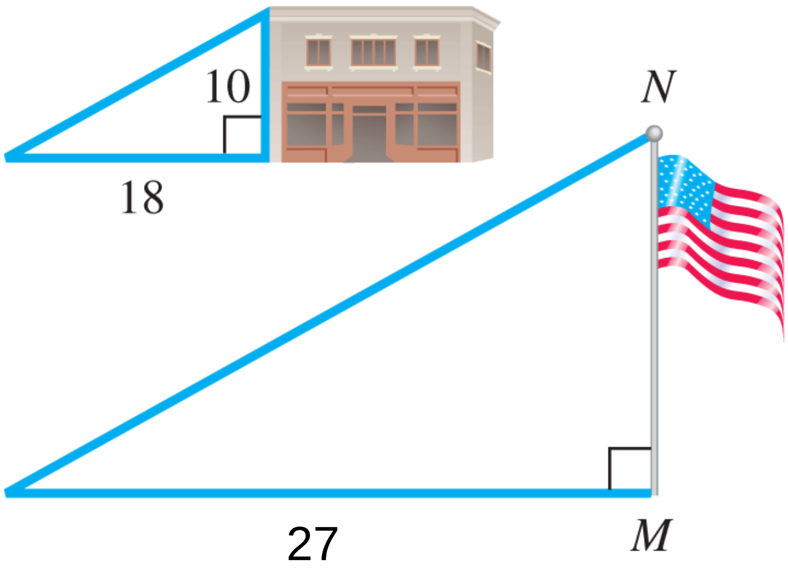
$$\frac{DE}{AC}=\frac{EF}{CB} \Rightarrow \frac{8}{16}=\frac{EF}{32}$$

$$\Rightarrow 8\cdot 32 = 16\cdot EF \Rightarrow 16 = EF$$

Side *EF* has length 16.

Example 5 FINDING THE HEIGHT OF A FLAGPOLE

Workers must measure the height of a building flagpole. They find that at the instant when the shadow of the station is 18 m long, the shadow of the flagpole is 27 m long. The building is 10 m high. Find the height of the flagpole.



The two triangles are similar, so corresponding sides are in proportion.

$$\frac{MN}{10} = \frac{27}{18}$$

Example 5 FINDING THE HEIGHT OF A FLAGPOLE (continued)

$$\frac{MN}{10} = \frac{3}{2}$$

Lowest terms

$$MN \cdot 2 = 10 \cdot 3$$

$$MN = 15$$

The flagpole is 15 m high.