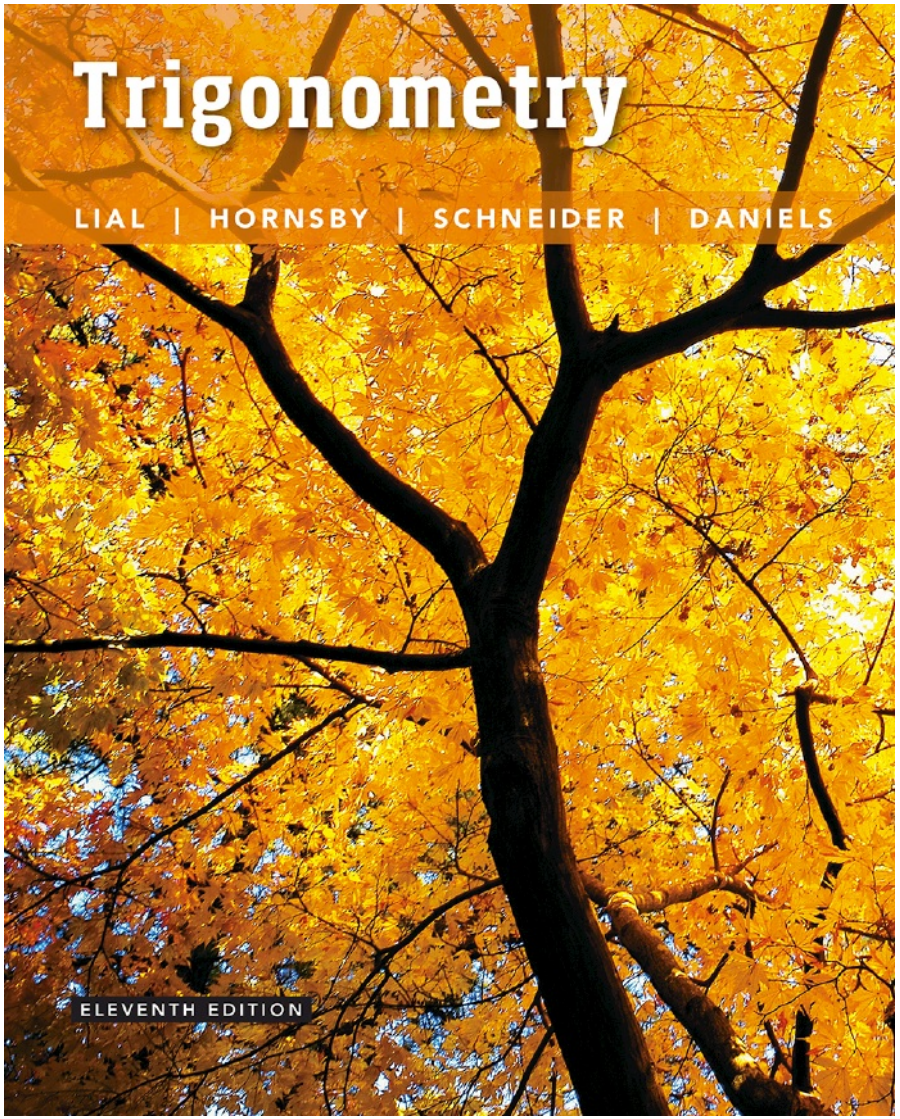
1

Trigonometric Functions

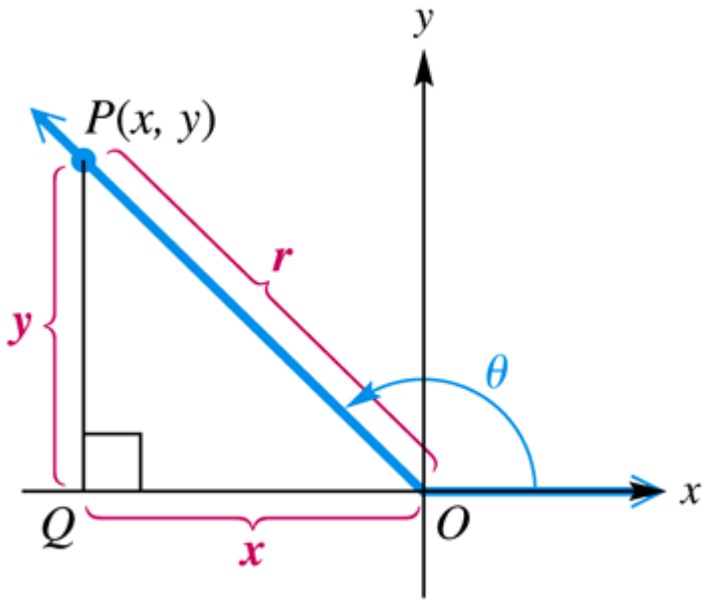


1.3 Trigonometric Functions

The Pythagorean Theorem and the Distance Formula ▪ Trigonometric Functions ▪ Quadrantal Angles ▪

Trigonometric Functions

Let (x, y) be a point other the origin on the terminal side of an angle θ in standard position. The distance from the point to the origin is $r = \sqrt{x^2 + y^2}$.



Trigonometric Functions

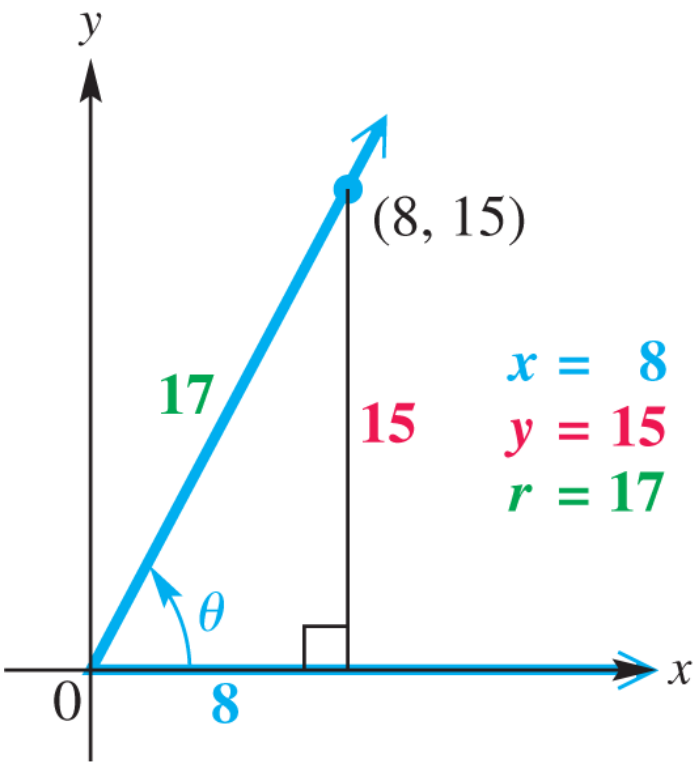
The six trigonometric functions of θ are defined as follows:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x},\ x \neq 0$$

$$\csc\theta = \frac{r}{y},\ y \neq 0 \qquad \sec\theta = \frac{r}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

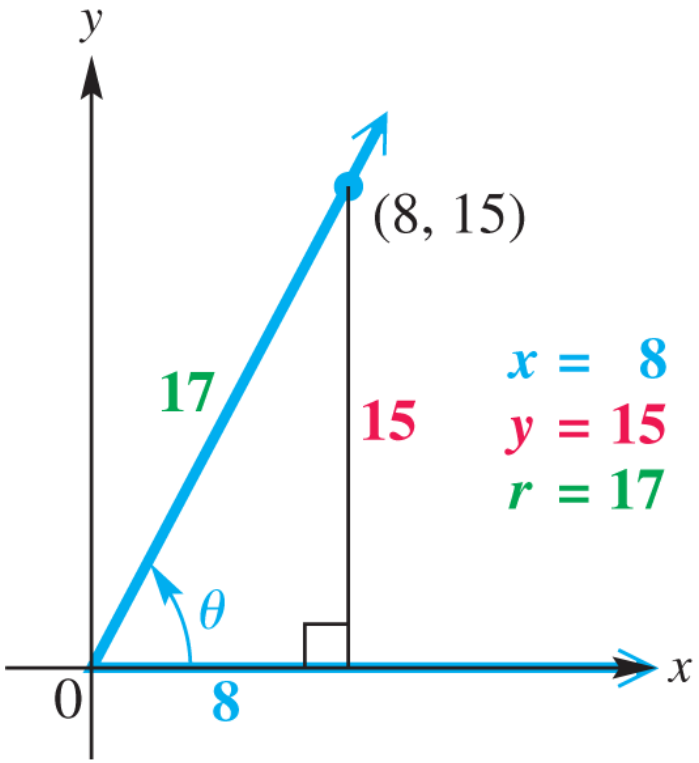
Example 1 FINDING FUNCTION VALUES OF AN ANGLE

The terminal side of angle θ in standard position passes through the point (8, 15). Find the values of the six trigonometric functions of angle θ.



The figure shows angle θ and the triangle formed by dropping a perpendicular from the point (8, 15) to the *x*-axis. The point (8, 15) is 8 units to the right of the *y*-axis and 15 units above the *x*-axis, so $x = 8$ and $y = 15$.

FINDING FUNCTION VALUES OF AN ANGLE (continued)



$$r = \sqrt{x^2 + y^2} = \sqrt{8^2 + 15^2}$$

$$= \sqrt{64 + 225} = \sqrt{289} = 17$$

We can now find the values of the six trigonometric functions of angle θ.

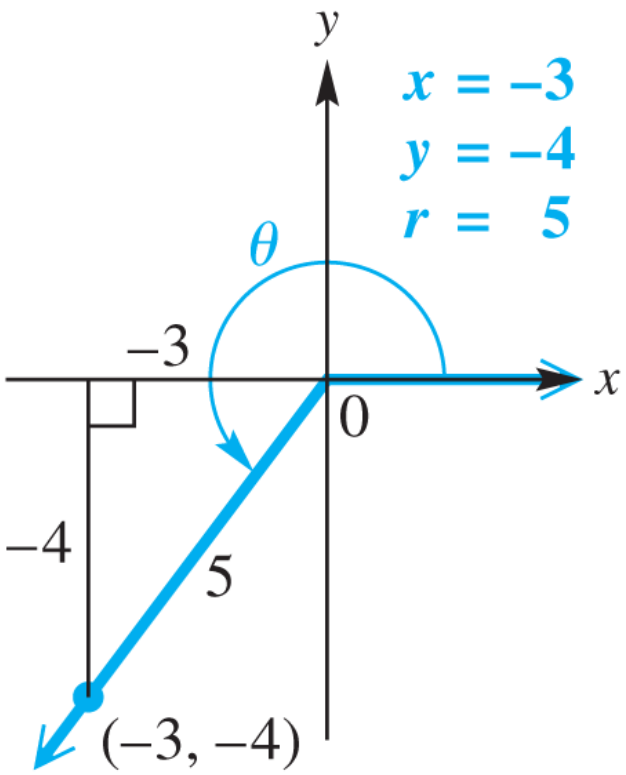
FINDING FUNCTION VALUES OF AN ANGLE (continued)

$$\sin\theta = \frac{y}{r} = \frac{15}{17} \qquad \cos\theta = \frac{x}{r} = \frac{8}{17} \qquad \tan\theta = \frac{y}{x} = \frac{15}{8}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{15} \qquad \sec\theta = \frac{r}{x} = \frac{17}{8} \qquad \cot\theta = \frac{x}{y} = \frac{8}{15}$$

Example 2 FINDING FUNCTION VALUES OF AN ANGLE

The terminal side of angle θ in standard position passes through the point (–3, –4). Find the values of the six trigonometric functions of angle θ.



$$r = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$$

FINDING FUNCTION VALUES OF AN ANGLE (continued)

Use the definitions of the trigonometric functions.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x},\ x \neq 0$$

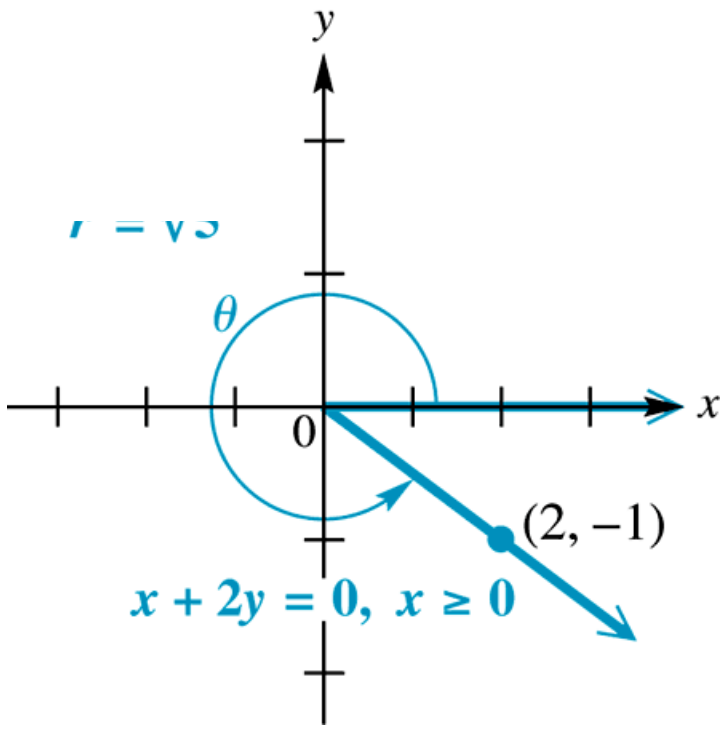
$$\csc\theta = \frac{r}{y},\ y \neq 0 \qquad \sec\theta = \frac{r}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

$$\sin\theta = \frac{-4}{5} = -\frac{4}{5} \qquad \cos\theta = \frac{-3}{5} = -\frac{3}{5} \qquad \tan\theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\csc\theta = \frac{5}{-4} = -\frac{5}{4} \qquad \sec\theta = \frac{5}{-3} = -\frac{5}{3} \qquad \cot\theta = \frac{-3}{-4} = \frac{3}{4}$$

Example 3 FINDING FUNCTION VALUES OF AN ANGLE

Find the six trigonometric function values of the angle θ in standard position, if the terminal side of θ is defined by $x + 2y = 0$, $x \geq 0$.



We can use any point except (0, 0) on the terminal side of θ to find the trigonometric function values.

Choose $x = 2$.

$$x + 2y = 0,\ x \geq 0$$

$$2 + 2y = 0$$

$$2y = -2 \Rightarrow y = -1$$

FINDING FUNCTION VALUES OF AN ANGLE (continued)

The point (2, –1) lies on the terminal side, and the corresponding value of r is $r = \sqrt{2^2 + (-1)^2} = \sqrt{5}$.

$$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{5}} = \frac{-1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominators.

$$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{y}{x} = -\frac{1}{2}$$

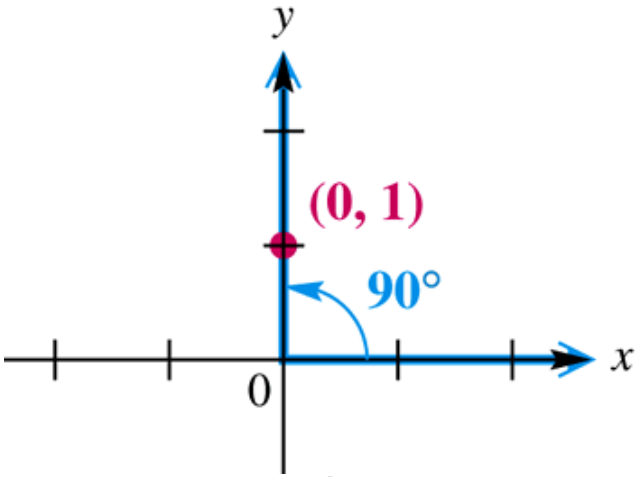
$$\cot\theta = \frac{x}{y} = -2$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{r}{y} = -\sqrt{5}$$

Example 4 FINDING FUNCTION VALUES OF QUADRANTAL ANGLES

(a) Find the values of the six trigonometric functions for an angle of 90°.



The terminal side passes through (0, 1). So $x = 0$, $y = 1$, and $r = 1$.

$$\sin 90° = \frac{1}{1} = 1$$

$$\cos 90° = \frac{0}{1} = 0$$

$$\tan 90° = \frac{1}{0}$$

undefined

$$\csc 90° = \frac{1}{1} = 1$$

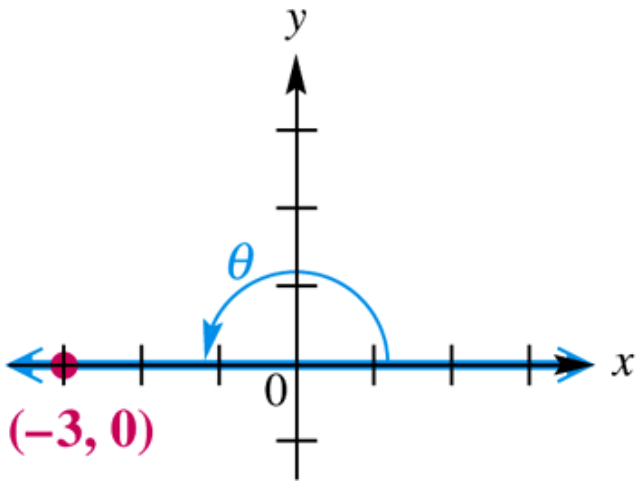
$$\sec 90° = \frac{1}{0}$$

undefined

$$\cot 90° = \frac{0}{1} = 0$$

Example 4 FINDING FUNCTION VALUES OF QUADRANTAL ANGLES

(b) Find the values of the six trigonometric functions for an angle θ in standard position with terminal side through $(-3, 0)$.



$x = -3$, $y = 0$, and $r = 3$.

$$\sin\theta = \frac{0}{3} = 0 \qquad \cos\theta = \frac{-3}{3} = -1 \qquad \tan\theta = \frac{0}{-3} = 0$$

$$\csc\theta = \frac{3}{0} \qquad \sec\theta = \frac{3}{-3} = -1 \qquad \cot\theta = \frac{-3}{0}$$

undefined undefined

Conditions for Undefined Function Values

Identify the terminal side of a quadrantal angle.

If the terminal side of the quadrantal angle lies along the *y*-axis, then the tangent and secant functions are undefined.

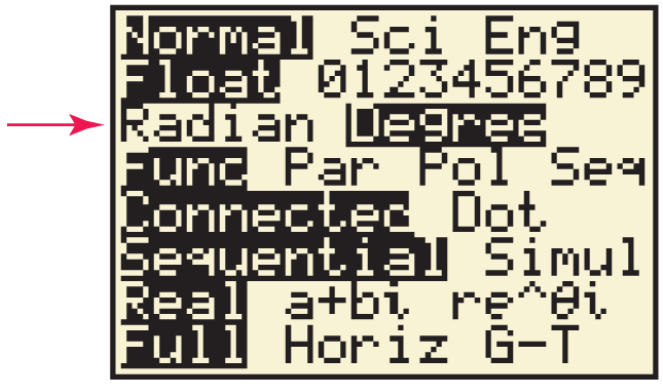
If the terminal side of a quadrantal angle lies along the *x*-axis, then the cotangent and cosecant functions are undefined.

Function Values of Quadrantal Angles

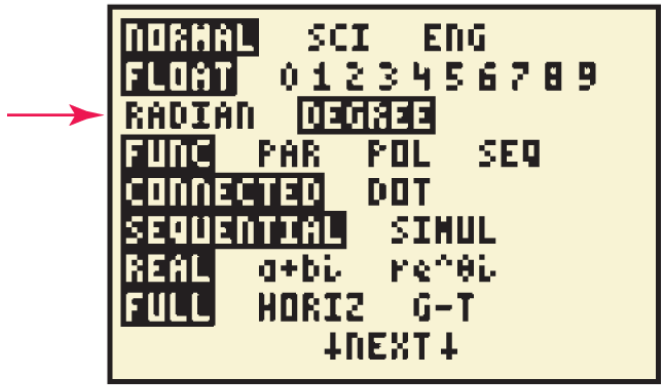
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
0°	0	1	0	Undefined	1	Undefined
90°	1	0	Undefined	0	Undefined	1
180°	0	−1	0	Undefined	−1	Undefined
270°	−1	0	Undefined	0	Undefined	−1
360°	0	1	0	Undefined	1	Undefined

Using a Calculator

Function values of quadrantal angles can be found with a calculator that has trigonometric function keys. ***Make sure the calculator is set in degree mode.***



TI-83 Plus



TI-84 Plus

Caution

One of the most common errors involving calculators in trigonometry occurs when the calculator is set for radian measure, rather than degree measure. Be sure you know how to set your calculator in degree mode.