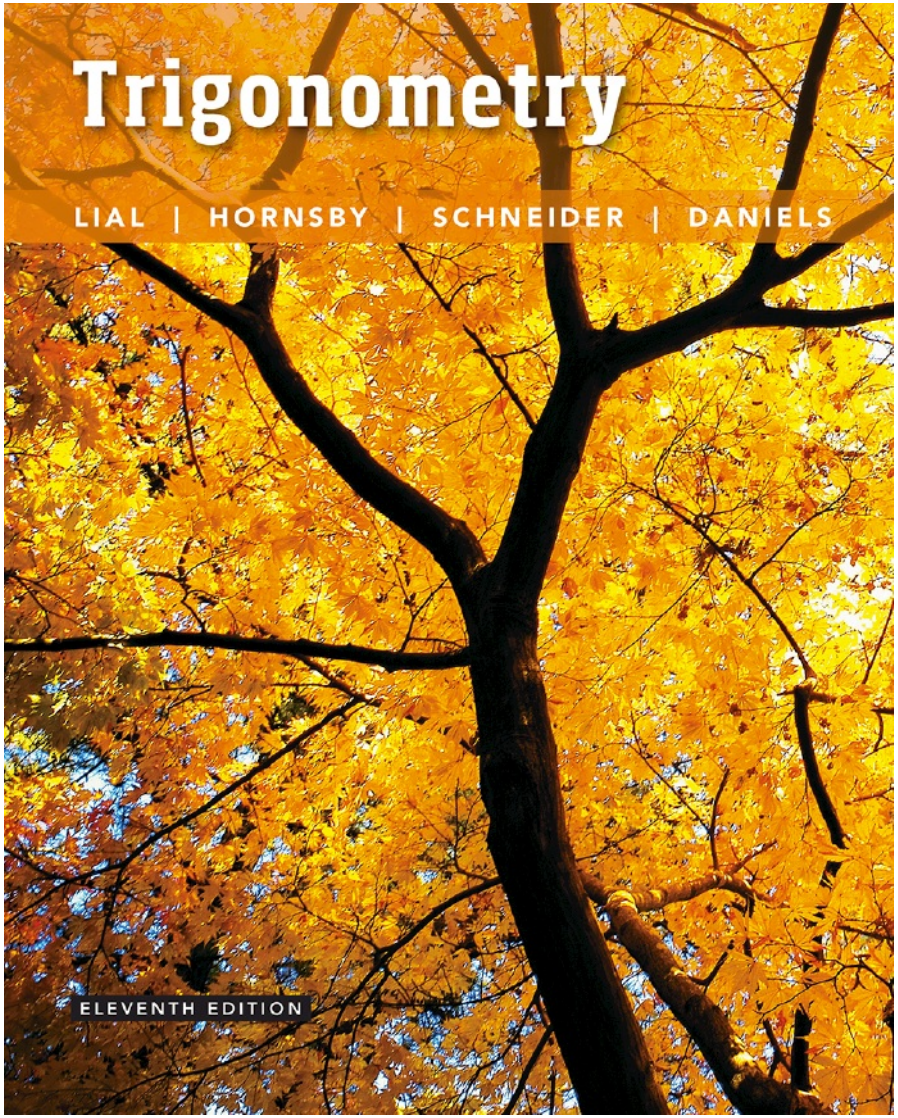
1

Trigonometric Functions



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1.4 Using the Definitions of the Trigonometric Functions

Reciprocal Identities ▪ Signs and Ranges of Function Values ▪ Pythagorean Identities ▪ Quotient Identities

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Reciprocal Identities

For all angles θ for which both functions are defined,

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

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Example 1(a) USING THE RECIPROCAL IDENTITIES

Find $\cos\theta$, given that $\sec\theta = \dfrac{5}{3}$.

Since cos *θ* is the reciprocal of sec *θ*,

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

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Example 1(b) USING THE RECIPROCAL IDENTITIES

Find $\sin\theta$, given that $\csc\theta = -\dfrac{\sqrt{12}}{2}$.

Since sin *θ* is the reciprocal of csc *θ*,

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{-\frac{\sqrt{12}}{2}} = -\frac{2}{\sqrt{12}}$$

$$= -\frac{2}{2\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$\sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$

$$= -\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

Rationalize the denominator.

$$= -\frac{\sqrt{3}}{3}$$

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Signs of Function Values

θ in Quadrant	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
I	+	+	+	+	+	+
II	+	−	−	−	−	+
III	−	−	+	+	−	−
IV	−	+	−	−	+	−

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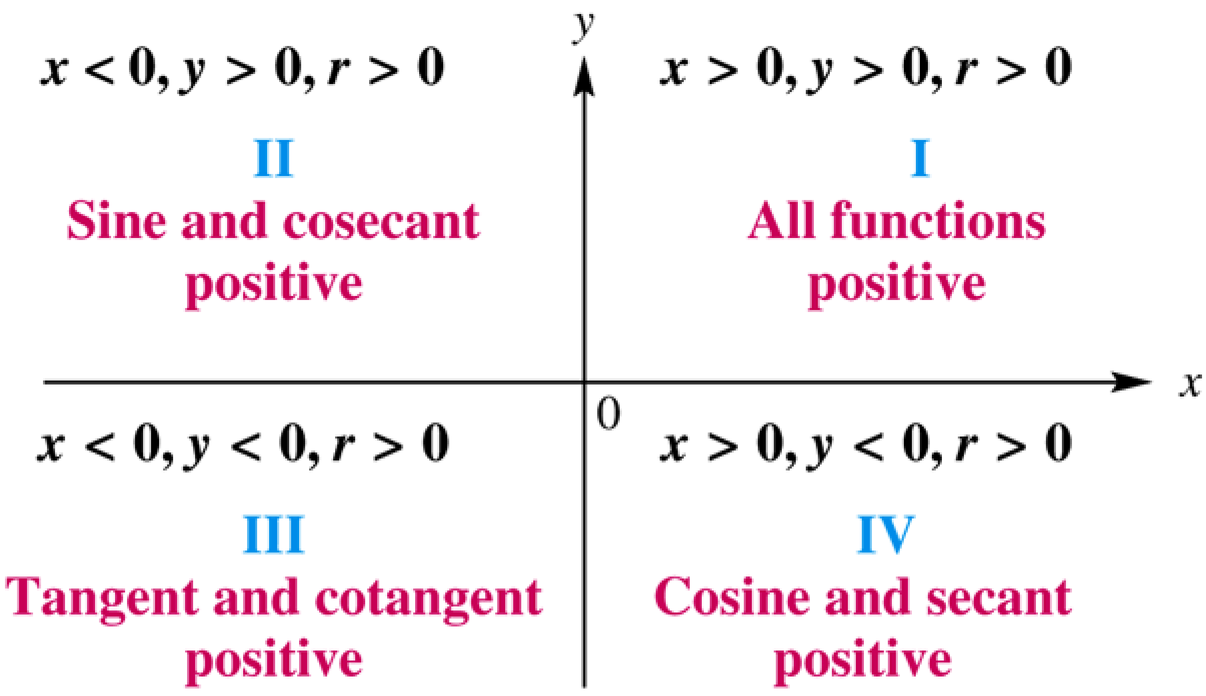
Signs of Function Values



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Example 2 DETERMINING SIGNS OF FUNCTIONS OF NONQUADRANTAL ANGLES

Determine the signs of the trigonometric functions of an angle in standard position with the given measure.

(a) $87°$

The angle lies in the first quadrant, so all of its trigonometric function values are positive.

(b) $300°$

The angle lies in quadrant IV, so the cosine and secant are positive, while the sine, cosecant, tangent, and cotangent are negative.

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Example 2 DETERMINING SIGNS OF FUNCTIONS OF NONQUADRANTAL ANGLES (cont.)

Determine the signs of the trigonometric functions of an angle in standard position with the given measure.

(c) –200°

The angle lies in quadrant II, so the sine and cosecant are positive, and all other function values are negative.

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Example 3 IDENTIFYING THE QUADRANT OF AN ANGLE

Identify the quadrant (or possible quadrants) of any angle θ that satisfies the given conditions.

(a) $\sin\theta > 0$, $\tan\theta < 0$.

Since $\sin\theta > 0$ in quadrants I and II, and $\tan\theta < 0$ in quadrants II and IV, both conditions are met only in quadrant II.

(b) $\cos\theta < 0$, $\sec\theta < 0$

The cosine and secant functions are both negative in quadrants II and III, so θ could be in either of these two quadrants.

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Ranges of Trigonometric Functions

Trigonometric Function of θ	Range (Set-Builder Notation)	Range (Interval Notation)
$\sin\theta$, $\cos\theta$	$\{y \mid \lvert y\rvert \le 1\}$	$[-1, 1]$
$\tan\theta$, $\cot\theta$	$\{y \mid y \text{ is a real number}\}$	$(-\infty, \infty)$
$\sec\theta$, $\csc\theta$	$\{y \mid \lvert y\rvert \ge 1\}$	$(-\infty, -1] \cup [1, \infty)$

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▶ Example 4 DECIDING WHETHER A VALUE IS IN THE RANGE OF A TRIGONOMETRIC FUNCTION

Decide whether each statement is *possible* or *impossible*.

(a) $\sin \theta = 2.5$

For any value of θ, we know that $-1 \leq \sin \theta \leq 1$. Since $2.5 > 1$, it is impossible to find a value of θ that satisfies $\sin \theta = 2.5$.

(b) $\tan \theta = 110.47$

The tangent function can take on any real number value. Thus, $\tan \theta = 110.47$ is possible.

(c) $\sec \theta = 0.6$

Since $|\sec \theta| \geq 1$ for all θ for which the secant is defined, the statement $\sec \theta = 0.6$ is impossible.

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Example 5 FINDING ALL FUNCTION VALUES GIVEN ONE VALUE AND THE QUADRANT

Suppose that angle θ is in quadrant II and $\sin\theta = \frac{2}{3}$. Find the values of the other five trigonometric functions.

Choose any point on the terminal side of angle θ.

$$\sin\theta = \frac{2}{3} = \frac{y}{r}$$

Let $r = 3$. Then $y = 2$.

$$x^2 + y^2 = r^2 \Rightarrow x^2 + 2^2 = 3^2 \Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5}$$

Since θ is in quadrant II, $x = -\sqrt{5}$.

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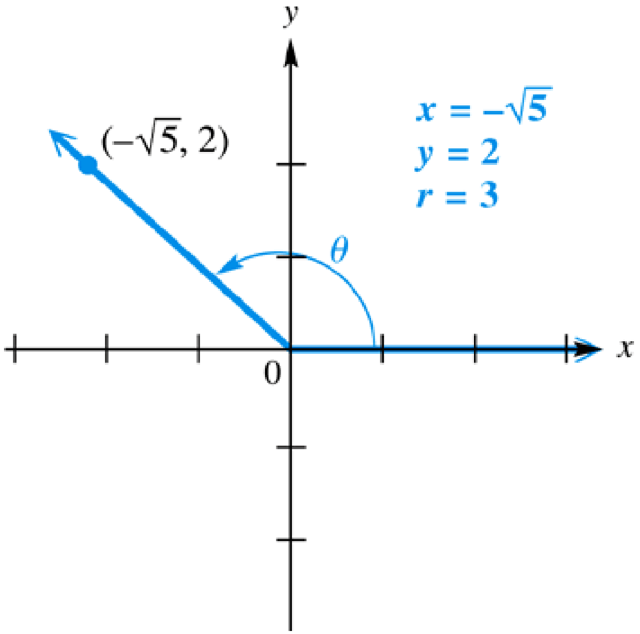
Example 5 FINDING ALL FUNCTION VALUES GIVEN ONE VALUE AND THE QUADRANT (continued)



$$\cos\theta = \frac{x}{r} = -\frac{\sqrt{5}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{3}{-\sqrt{5}}$$

$$= -\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

Remember to rationalize the denominator.

$$\tan\theta = \frac{y}{x} = \frac{2}{-\sqrt{5}}$$

$$= -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

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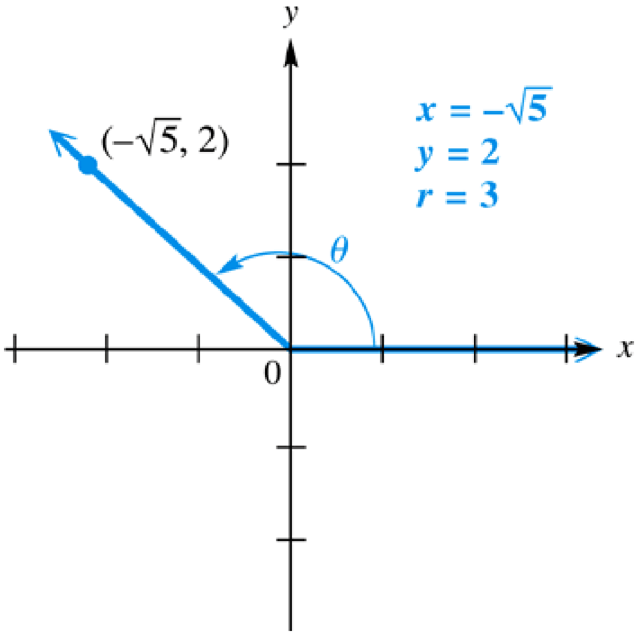
Example 5 FINDING ALL FUNCTION VALUES GIVEN ONE VALUE AND THE QUADRANT (continued)



$$\cot\theta = \frac{x}{y} = -\frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{3}{2}$$

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Pythagorean Identities

For all angles θ for which the function values are defined, the following identities hold.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

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Quotient Identities

For all angles θ for which the denominators are not zero, the following identities hold.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

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Example 6 USING IDENTITIES TO FIND FUNCTION VALUES

Find $\sin\theta$ and $\tan\theta$, given that $\cos\theta = -\frac{\sqrt{3}}{4}$ and $\sin\theta > 0$.

Start with $\sin^2\theta + \cos^2\theta = 1$.

$$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1$$

$$\sin^2\theta = \frac{13}{16}$$

$$\sin\theta = \pm\sqrt{\frac{13}{16}} = \pm\frac{\sqrt{13}}{4}$$

$$\sin\theta = \frac{\sqrt{13}}{4}$$

Choose the positive square root since sin θ >0.

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Example 6 USING IDENTITIES TO FIND FUNCTION VALUES (continued)

To find tan θ, use the quotient identity $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}} = -\frac{\sqrt{13}}{\sqrt{3}}$$

$$= -\frac{\sqrt{13}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{39}}{3}$$

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Caution

Be careful to choose the correct sign when square roots are taken.

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Example 7 USING IDENTITIES TO FIND FUNCTION VALUES

Find sin θ and cos θ, given that $\tan\theta = \frac{4}{3}$ and θ is in quadrant III.

Since θ is in quadrant III, sin θ and cos θ will both be negative. It is tempting to say that since

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4}{3} \quad \text{and} \quad \tan\theta = \frac{4}{3}$$

then sin θ = –4 and cos θ = –3. This is *incorrect*, however, since both sin θ and cos θ must be in the interval [–1,1].

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Example 7 USING IDENTITIES TO FIND FUNCTION VALUES (continued)

Use the identity $\tan^2\theta + 1 = \sec^2\theta$ to find sec *θ*. Then use the reciprocal identity to find cos *θ*.

$$\left(\frac{4}{3}\right)^2 + 1 = \sec^2\theta$$

$$\frac{25}{9} = \sec^2\theta$$

$$-\frac{5}{3} = \sec\theta$$

Choose the negative square root since sec *θ* <0 when *θ* is in quadrant III.

$$-\frac{3}{5} = \cos\theta$$

Secant and cosine are reciprocals.

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Example 7 USING IDENTITIES TO FIND FUNCTION VALUES (continued)

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(-\frac{3}{5}\right)^2$$

$$\sin^2\theta = \frac{16}{25}$$

$$\sin\theta = -\frac{4}{5}$$

Choose the negative square root since $\sin\theta < 0$ for θ in quadrant III.

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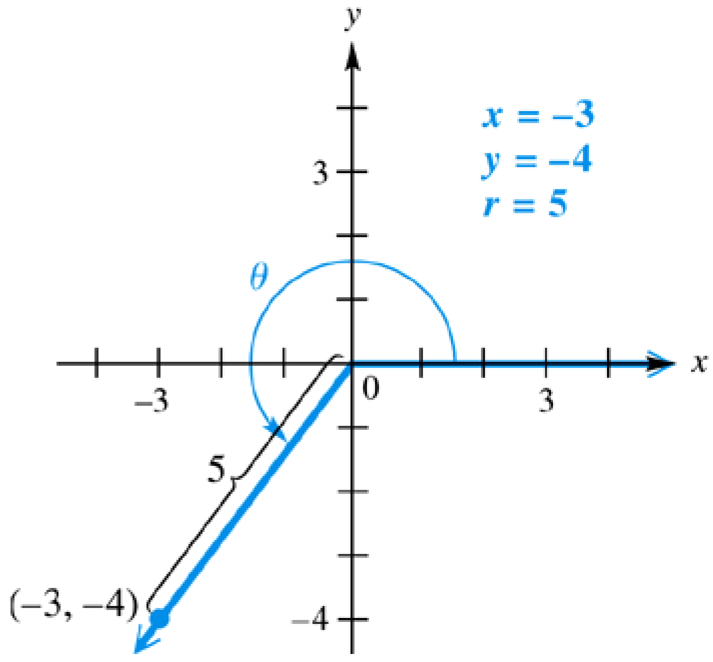
Example 7 **USING IDENTITIES TO FIND FUNCTION VALUES (continued)**

This example can also be worked by sketching θ in standard position in quadrant III, finding r to be 5, and then using the definitions of sin θ and cos θ in terms of x, y, and r.



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