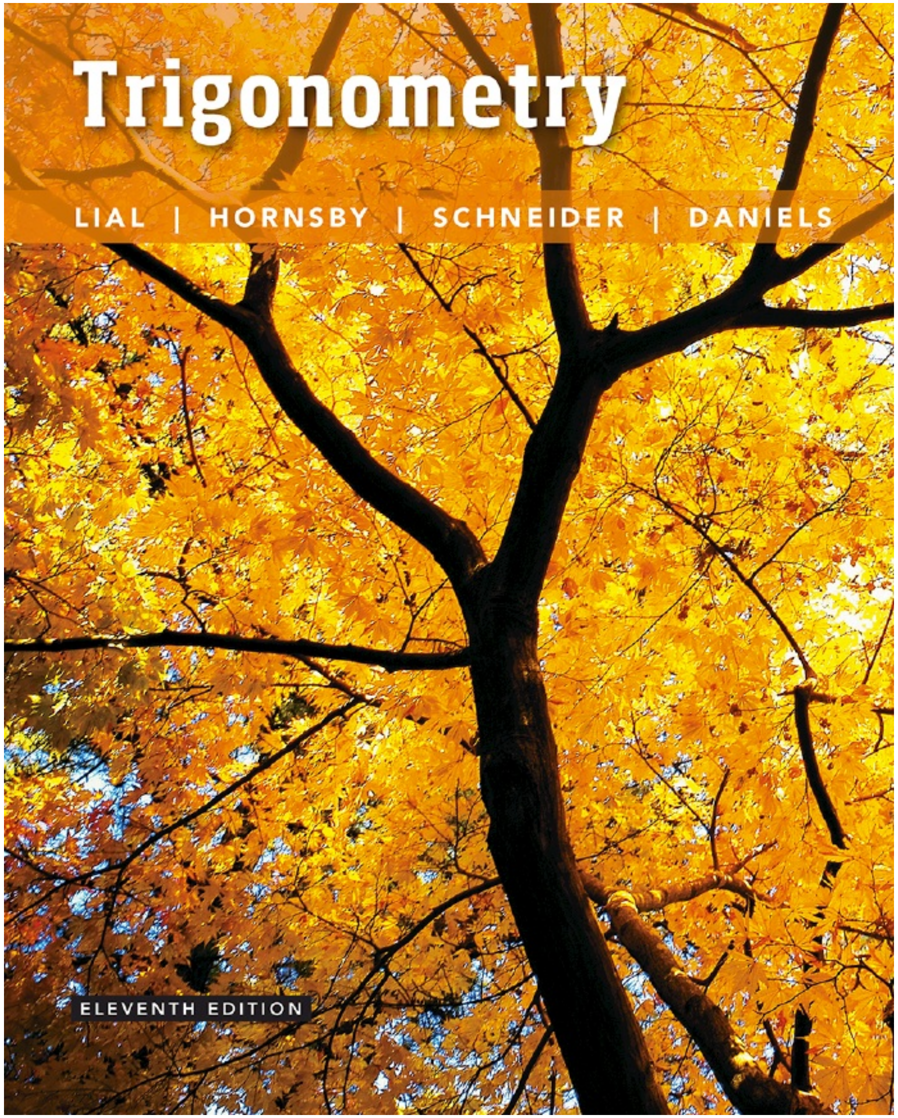
2

Acute Angles and Right Triangles



2.1 Trigonometric Functions of Acute Angles

Right-Triangle-Based Definitions of the Trigonometric Functions ▪ Cofunctions ▪ How Function Values Change as Angle Change ▪ Trigonometric Function Values of Special Angles

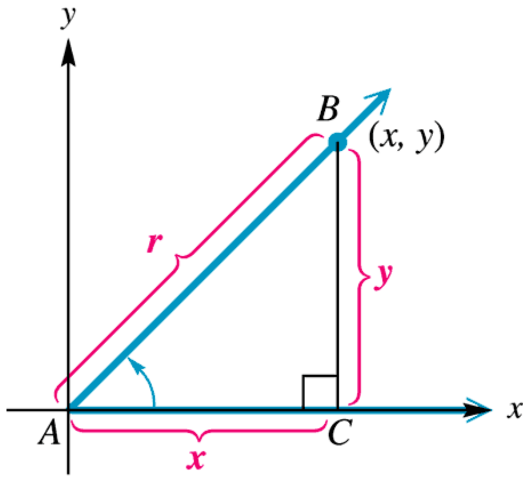
Right-Triangle-Based Definitions of Trigonometric Functions

Let A represent any acute angle in standard position.

$$\sin A = \frac{y}{r} = \frac{\text{side opposite } A}{\text{hypotenuse}} \qquad \csc A = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite } A}$$

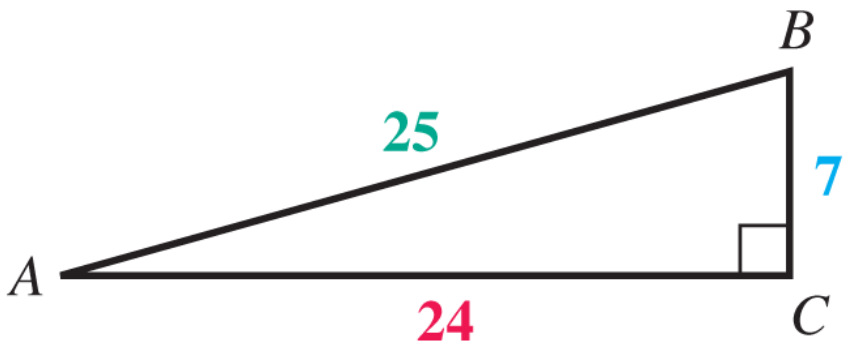
$$\cos A = \frac{x}{r} = \frac{\text{side adjacent to } A}{\text{hypotenuse}} \qquad \sec A = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent to } A}$$

$$\tan A = \frac{y}{x} = \frac{\text{side opposite } A}{\text{side adjacent to } A} \qquad \cot A = \frac{x}{y} = \frac{\text{side adjacent to } A}{\text{side opposite } A}$$



Example 1 FINDING TRIGONOMETRIC FUNCTION VALUES OF AN ACUTE ANGLE

Find the sine, cosine, and tangent values for angles *A* and *B.*



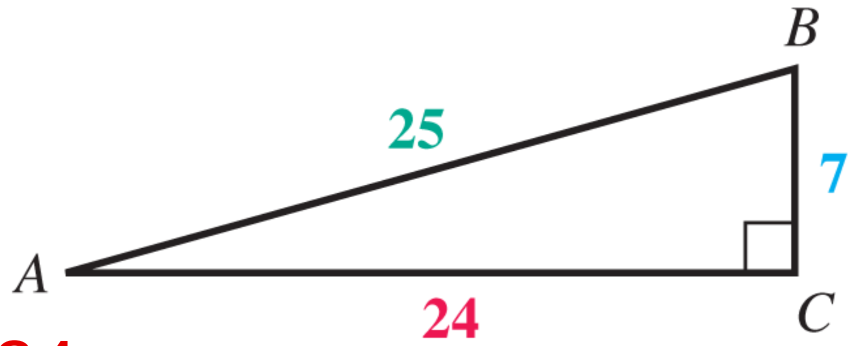
$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{7}{25}$$

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{7}{24}$$

Example 1 FINDING TRIGONOMETRIC FUNCTION VALUES OF AN ACUTE ANGLE (cont.)

Find the sine, cosine, and tangent values for angles *A* and *B.*



$$\sin B = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\cos B = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{7}{25}$$

$$\tan B = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{24}{7}$$

Cofunction Identities

For any acute angle A, cofunction values of complementary angles are equal.

$\sin A = \cos(90° - A)$ $\cos A = \sin(90° - A)$

$\tan A = \cot(90° - A)$ $\cot A = \tan(90° - A)$

$\sec A = \csc(90° - A)$ $\csc A = \sec(90° - A)$

Example 2 WRITING FUNCTIONS IN TERMS OF COFUNCTIONS

Write each function in terms of its cofunction.

(a) $\cos 52° = \sin (90° – 52°) = \sin 38°$

(b) $\tan 71° = \cot (90° – 71°) = \cot 19°$

(c) $\sec 24° = \csc (90° – 24°) = \csc 66°$

Example 3 SOLVING EQUATIONS USING COFUNCTION IDENTITIES

Find one solution for the equation. Assume all angles involved are acute angles.

(a) $\cos(\theta + 4°) = \sin(3\theta + 2°)$

Since sine and cosine are cofunctions, the equation is true if the sum of the angles is 90º.

$$(\theta + 4°) + (3\theta + 2°) = 90°$$

$$4\theta + 6° = 90°$$ Combine like terms.

$$4\theta = 84°$$ Subtract 6°.

$$\theta = 21°$$ Divide by 4.

Example 3 SOLVING EQUATIONS USING COFUNCTION IDENTITIES (continued)

Find one solution for the equation. Assume all angles involved are acute angles.

(b) $\tan(2\theta - 18°) = \cot(\theta + 18°)$

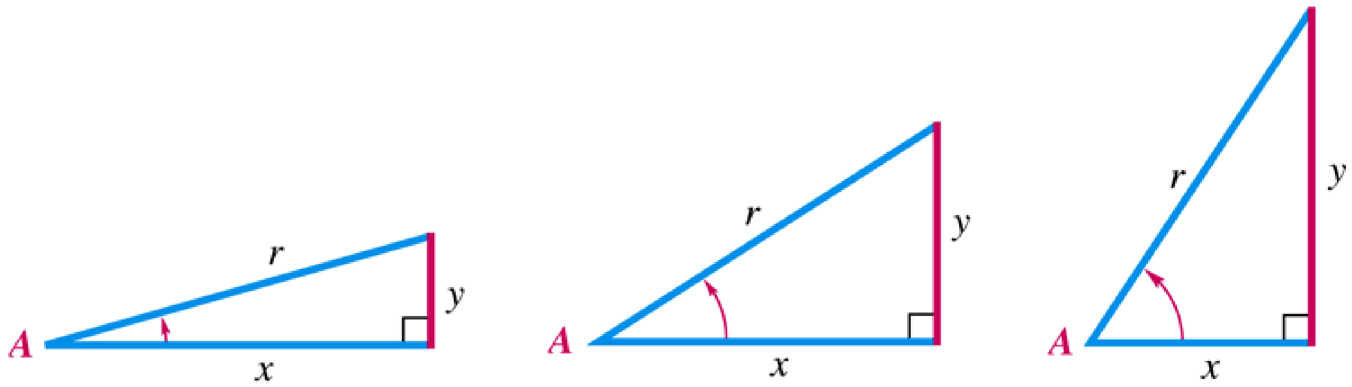
Since tangent and cotangent are cofunctions, the equation is true if the sum of the angles is 90º.

$$(2\theta - 18°) + (\theta + 18°) = 90°$$

$$3\theta = 90°$$

$$\theta = 30°$$

Increasing/Decreasing Functions



$\sin A = \frac{y}{r}$

As *A* increases, *y* increases and *x* decreases.

Since *r* is fixed,

sin *A* increases	csc *A* decreases
cos *A* decreases	sec *A* increases
tan *A* increases	cot *A* decreases

Example 4 COMPARING FUNCTION VALUES OF ACUTE ANGLES

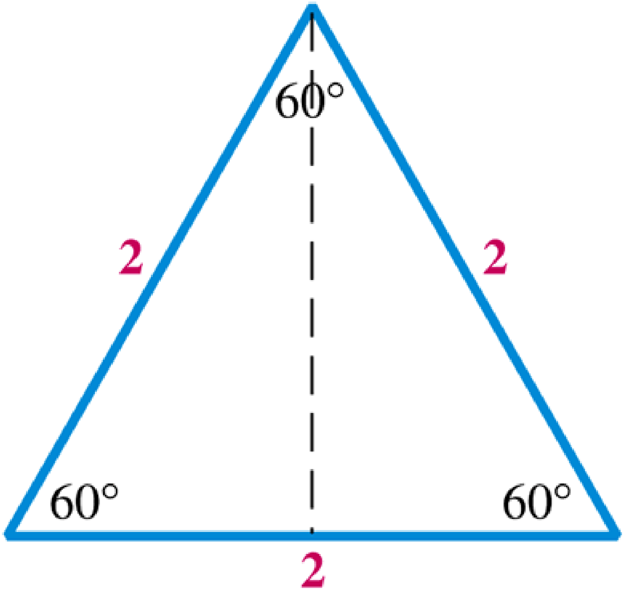
Determine whether each statement is *true* or *false*.

(a) $\sin 21° > \sin 18°$ (b) $\sec 56° \leq \sec 49°$

(a) In the interval from 0° to 90°, as the angle increases, so does the sine of the angle, which makes $\sin 21° > \sin 18°$ a true statement.

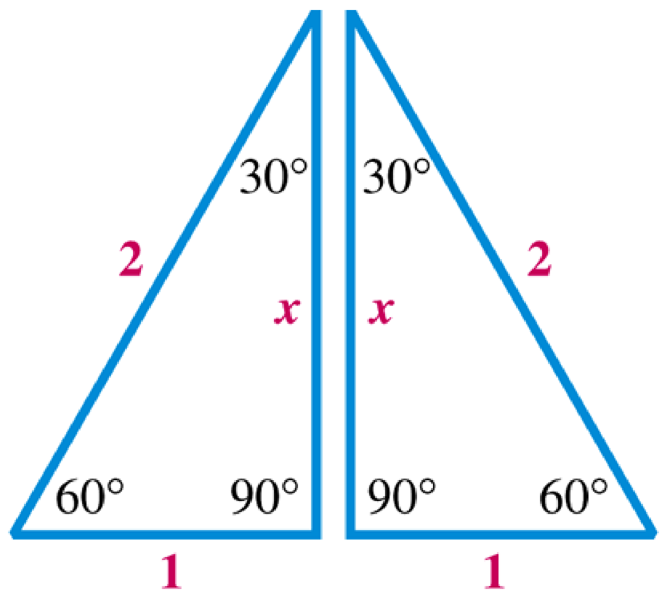
(b) For fixed *r*, increasing an angle from 0° to 90°, causes *x* to decrease. Therefore, $\sec \theta$ increases. The given statement, $\sec 56° \leq \sec 49°$, is false.

30°- 60° Triangles



Equilateral triangle

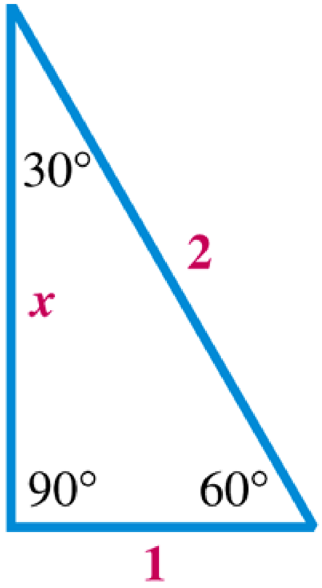
Bisect one angle of an equilateral triangle to create two 30°-60° triangles.



30°–60° right triangle

30°- 60° Triangles

Use the Pythagorean theorem to solve for *x*.



$$2^2 = 1^2 + x^2$$
$$4 = 1 + x^2$$
$$3 = x^2$$
$$\sqrt{3} = x$$

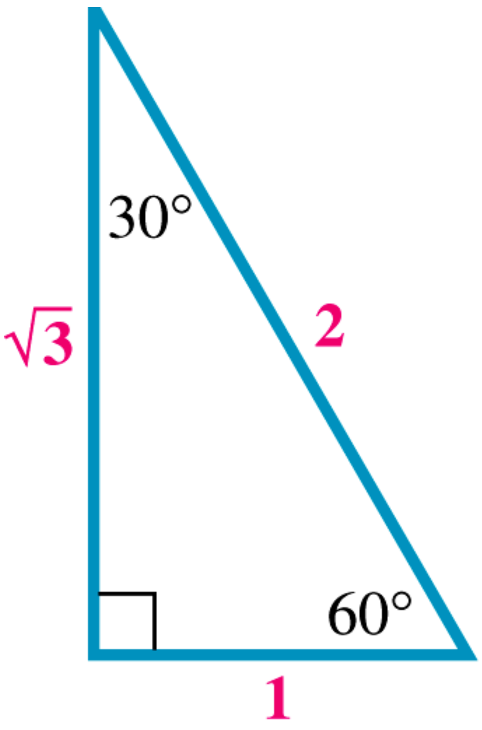
▶ Example 5 FINDING TRIGONOMETRIC FUNCTION VALUES FOR 60°

Find the six trigonometric function values for a 60° angle.

$$\sin 60° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\tan 60° = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$



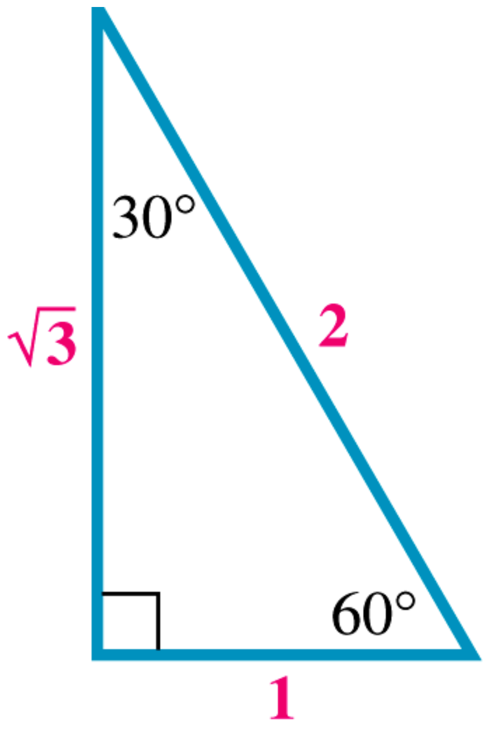
Example 5 FINDING TRIGONOMETRIC FUNCTION VALUES FOR 60° (continued)

Find the six trigonometric function values for a 60° angle.

$$\cot 60° = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec 60° = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{2}{1} = 2$$

$$\csc 60° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$



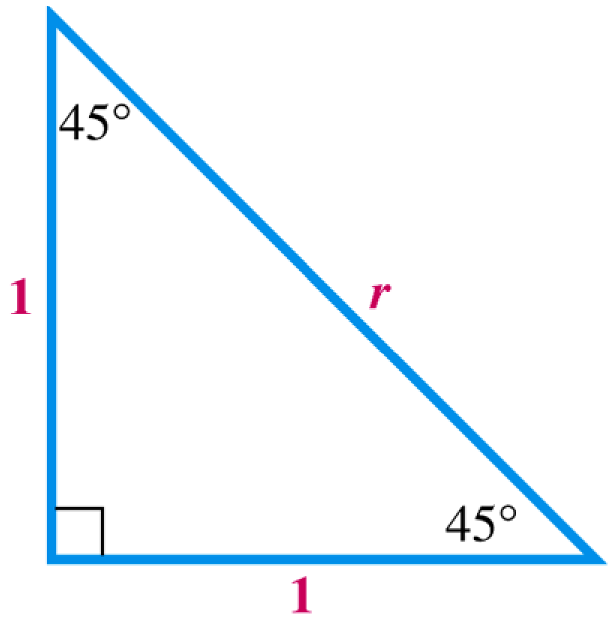
45°- 45° Right Triangles

Use the Pythagorean theorem to solve for *r*.

$$1^2 + 1^2 = r^2$$

$$2 = r^2$$

$$\sqrt{2} = r$$



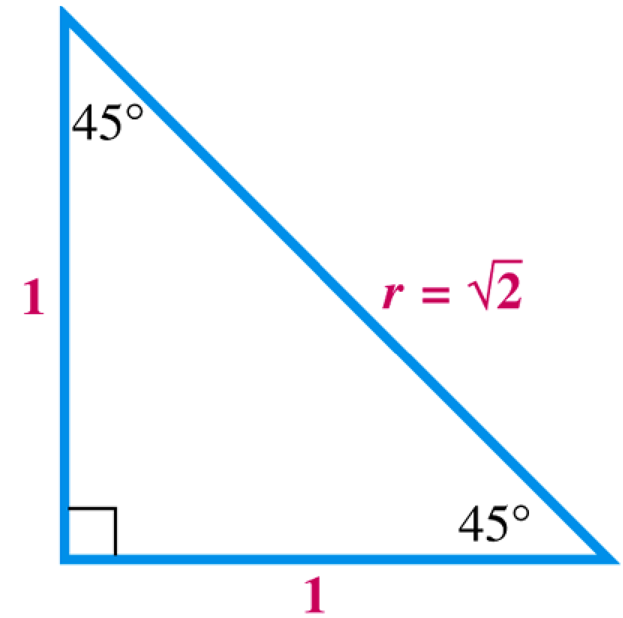
45°– 45° right triangle

45°- 45° Right Triangles

$$\sin 45° = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45° = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{1}{1} = 1$$



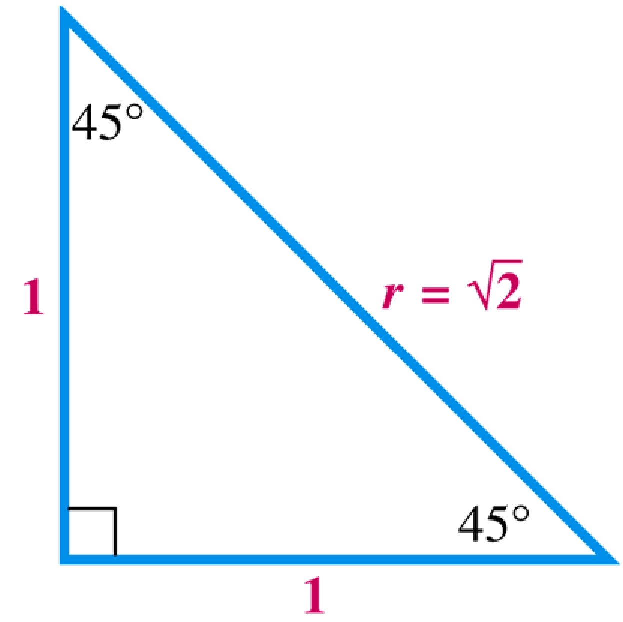
45°– 45° right triangle

45°- 45° Right Triangles

$$\cot 45° = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{1}{1} = 1$$

$$\sec 45° = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\csc 45° = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$



45°– 45° right triangle

Function Values of Special Angles

θ	sin θ	cos θ	tan θ	cot θ	sec θ	csc θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$