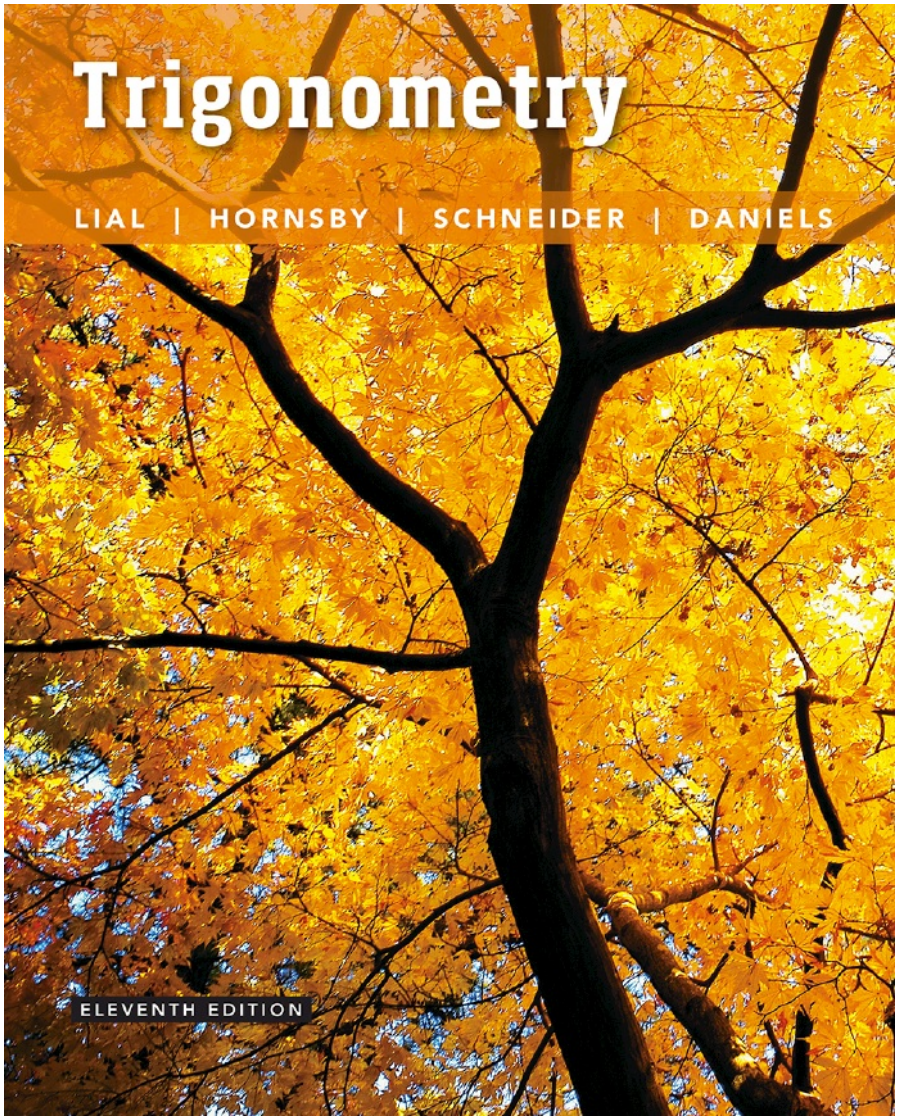
3

Radian Measure and the Unit Circle

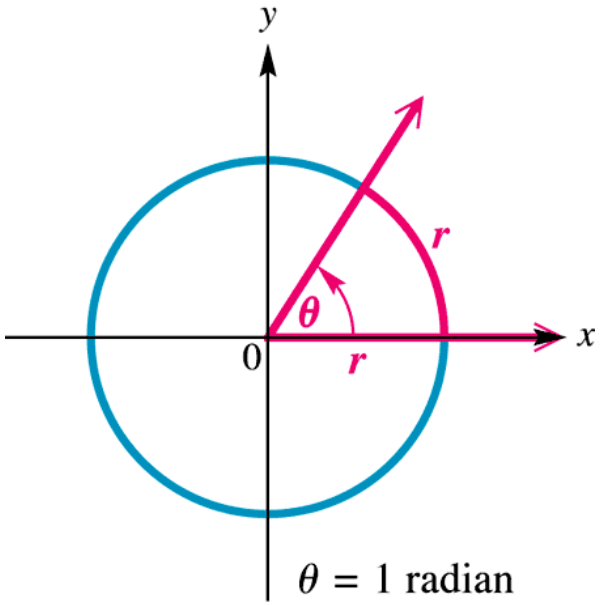


3.1 Radian Measure

Radian Measure ▪ Converting between Degrees and Radians ▪ Trigonometric Function Values of Angles in Radians

Radian

An angle with its vertex at the center of a circle that intercepts an arc on the circle equal in length to the radius of the circle has a measure of **1 radian.**



Converting Between Degrees and Radians

Multiply a degree measure by $\frac{\pi}{180}$ radian and simplify to convert to radians.

Multiply a radian measure by $\frac{180^\circ}{\pi}$ and simplify to convert to degrees.

CONVERTING DEGREES TO RADIANS

Convert each degree measure to radians.

(a) $45° = 45\left(\frac{\pi}{180} \text{ radian}\right) = \frac{\pi}{4} \text{ radian}$

(b) $-270° = -270\left(\frac{\pi}{180} \text{ radian}\right) = -\frac{3\pi}{2} \text{ radian}$

(c) $249.8° = 249.8\left(\frac{\pi}{180} \text{ radian}\right) \approx 4.360 \text{ radians}$

CONVERTING RADIANS TO DEGREES

Convert each radian measure to degrees.

(a) $\frac{9\pi}{4}$ radians $= \frac{9\pi}{4}\left(\frac{180^\circ}{\pi}\right) = 405^\circ$

(b) $-\frac{5\pi}{6}$ radians $= -\frac{5\pi}{6}\left(\frac{180^\circ}{\pi}\right) = -150^\circ$

(c) 4.25 radians $= 4.25\left(\frac{180^\circ}{\pi}\right) \approx 253.5^\circ = 243^\circ 30'$

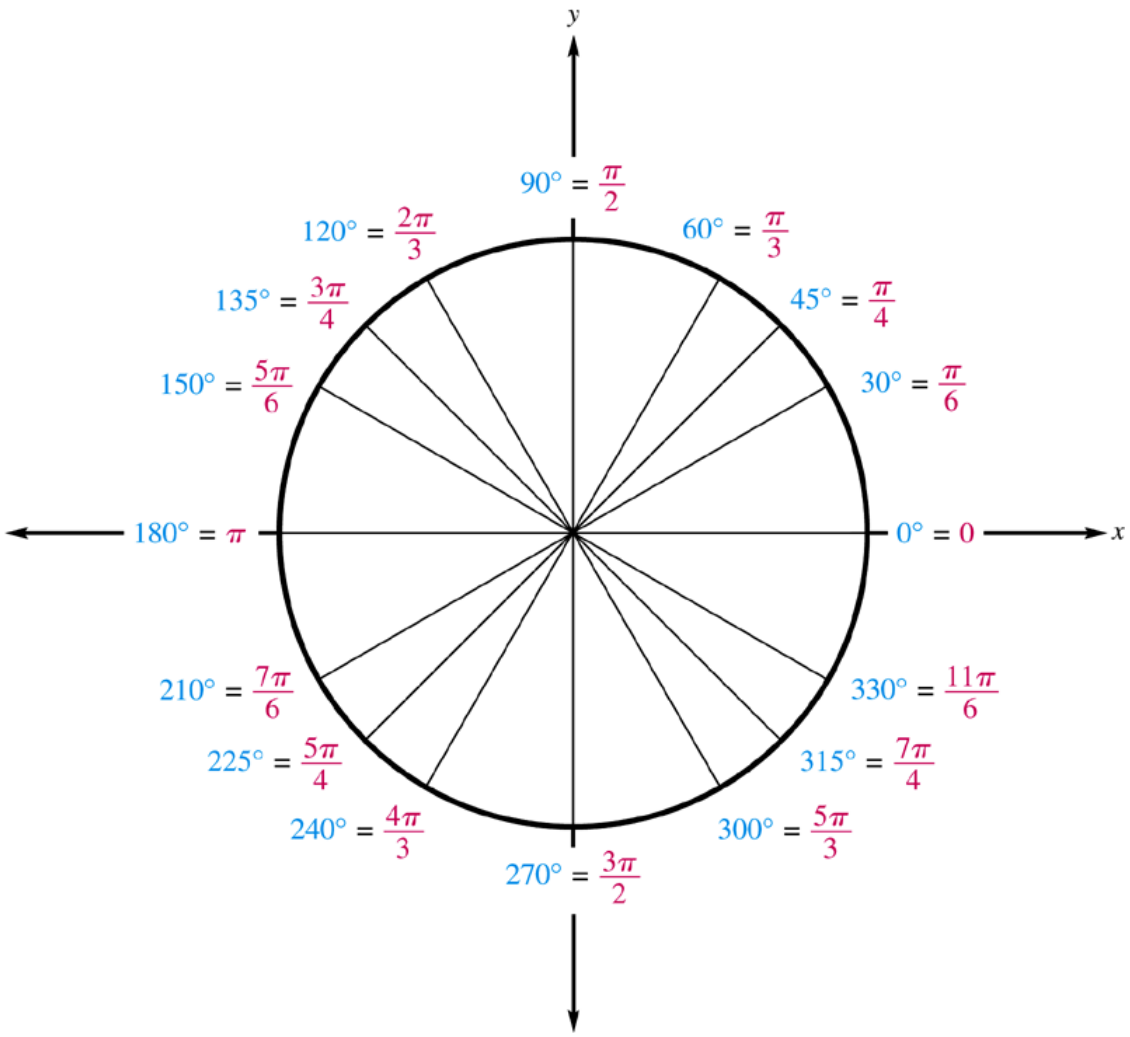
Agreement on Angle Measurement Units

If no unit of angle measure is specified, then the angle is understood to be measured in radians.

Equivalent Angle Measures in Degrees and Radians

Degrees	Radians		Degrees	Radians	
	Exact	Approximate		Exact	Approximate
0°	0	0	90°	$\frac{\pi}{2}$	1.57
30°	$\frac{\pi}{6}$	.52	180°	π	3.14
45°	$\frac{\pi}{4}$	.79	270°	$\frac{3\pi}{2}$	4.71
60°	$\frac{\pi}{3}$	1.05	360°	2π	6.28

Equivalent Angle Measures in Degrees and Radians



FINDING FUNCTION VALUES OF ANGLES IN RADIAN MEASURE

Find each function value.

(a) $\tan\frac{2\pi}{3} = \tan\left(\frac{2\pi}{3}\cdot\frac{180^\circ}{\pi}\right) = \tan 120^\circ = -\sqrt{3}$

(b) $\sin\frac{3\pi}{2} = \sin 270^\circ = -1$

(c) $\cos\left(-\frac{4\pi}{3}\right) = \cos\left(-\frac{4\pi}{3}\cdot\frac{180^\circ}{\pi}\right)$

$= -\cos 60^\circ = -\frac{1}{2}$