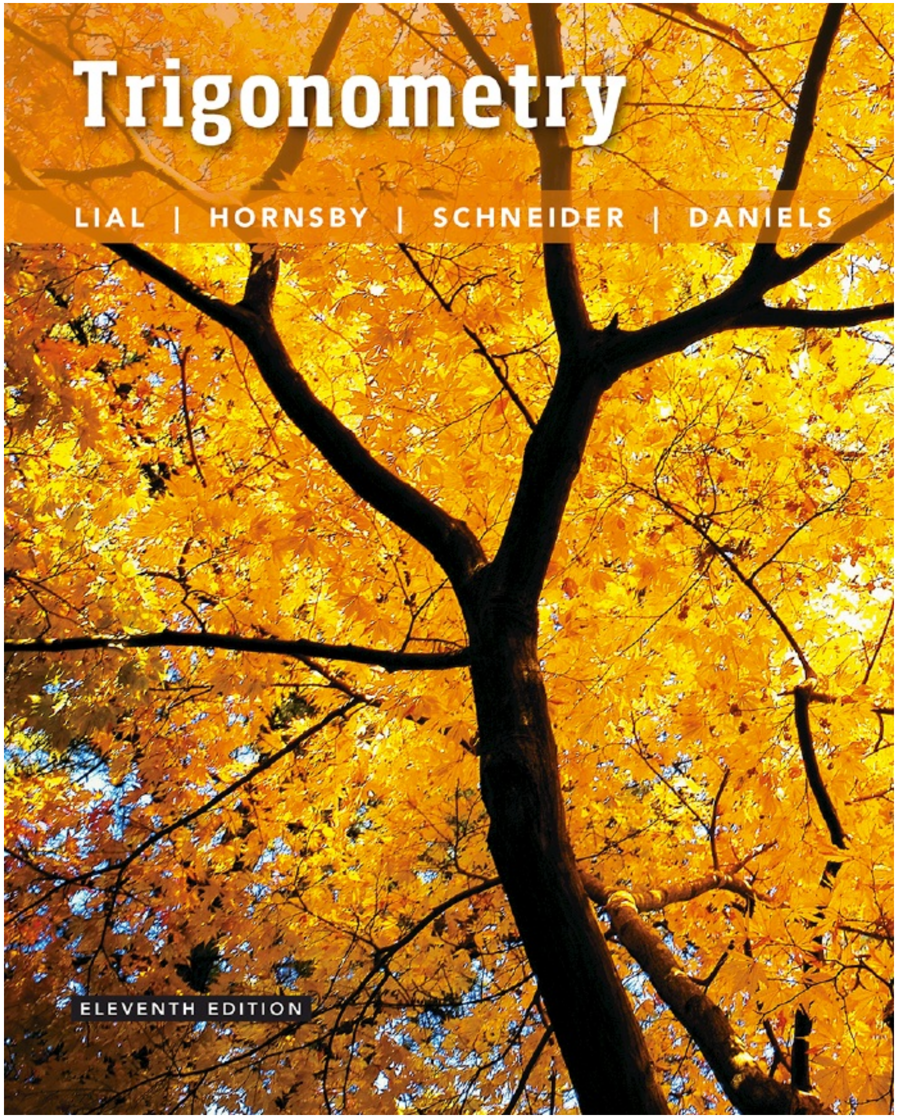
3

Radian Measure and the Unit Circle



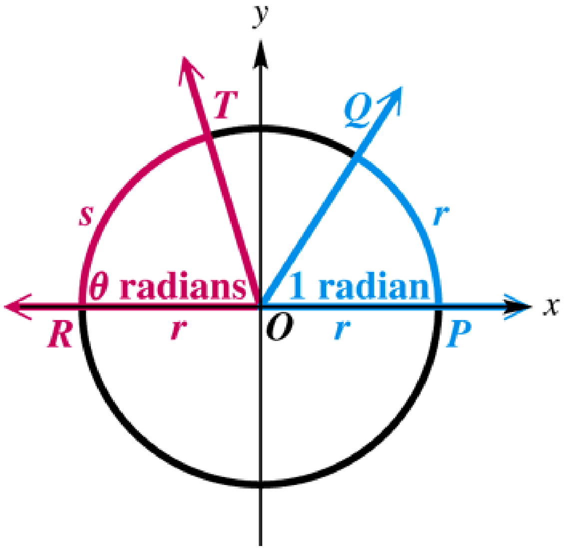
3.2 Applications of Radian Measure

Arc Length on a Circle ▪ Area of a Sector of a Circle

Arc Length

The length *s* of the arc intercepted on a circle of radius *r* by a central angle of measure θ radians is given by the product of the radius and the radian measure of the angle.

$$s = r\theta, \quad \text{where } \theta \text{ is in radians}$$



Caution

When the formula $s = r\theta$ is applied, the value of θ **MUST** ***be expressed in radians, not degrees.***

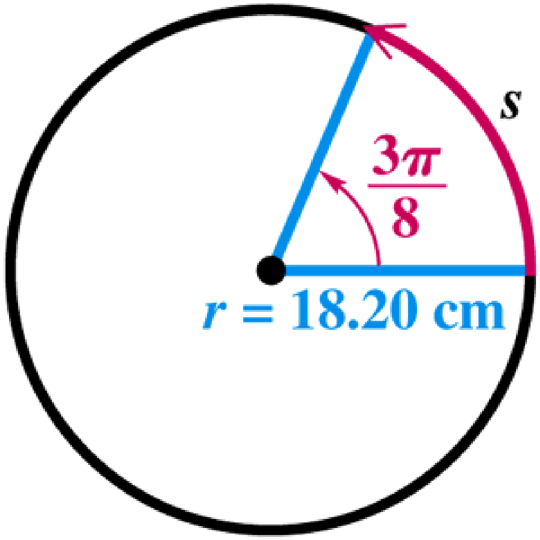
Example 1(a) FINDING ARC LENGTH USING $s = r\theta$

A circle has radius 18.20 cm. Find the length of the arc intercepted by a central angle with measure $\frac{3\pi}{8}$ radians.

$$s = r\theta$$

$$s = 18.20\left(\frac{3\pi}{8}\right)\text{cm}$$

$$s \approx 21.44 \text{ cm}$$



Example 1(b) FINDING ARC LENGTH USING $s = r\theta$

A circle has radius 18.20 cm. Find the length of the arc intercepted by a central angle with measure 144°.

Convert θ to radians.

$$144° = 144\left(\frac{\pi}{180}\right)$$

$$= \frac{4\pi}{5} \text{ radians}$$

$$s = r\theta$$

$$s = 18.20\left(\frac{4\pi}{5}\right)$$

$$\approx 45.74 \text{ cm}$$

Example 2 FINDING THE DISTANCE BETWEEN TWO CITIES

Latitude gives the measure of a central angle with vertex at Earth's center whose initial side goes through the equator and whose terminal side goes through the given location. Reno, Nevada, is approximately due north of Los Angeles. The latitude of Reno is 40° N, and that of Los Angeles is 34° N. (The N in 34° N means *north* of the equator.) The radius of Earth is about 6400 km. Find the north-south distance between the two cities.

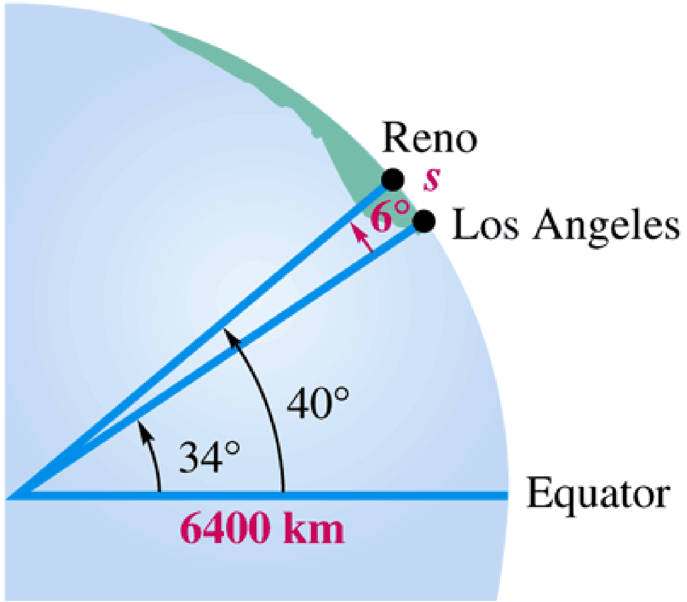
Example 2 FINDING THE DISTANCE BETWEEN TWO CITIES (continued)

The central angle between Reno and Los Angeles is 40° – 34° = 6°. Convert 6° to radians:

$$6^\circ = 6\left(\frac{\pi}{180}\right) = \frac{\pi}{30} \text{ radian}$$

The distance between the two cities is given by *s*.

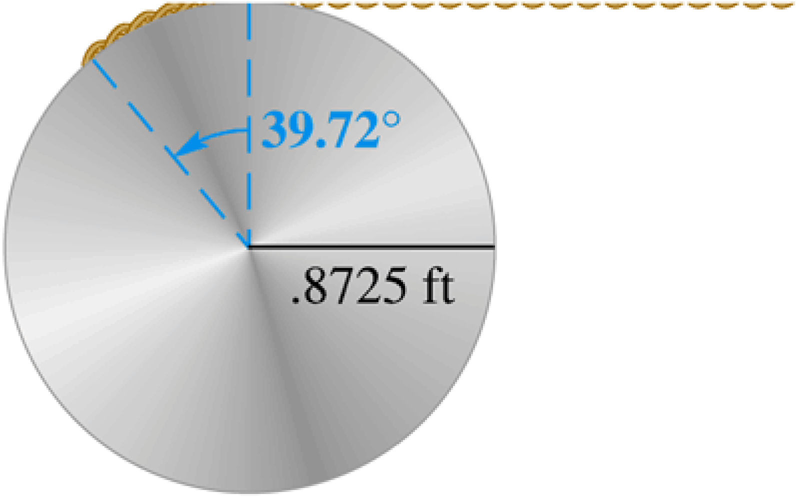
$$s = r\theta = 6400\left(\frac{\pi}{30}\right) \approx 670 \text{ km}$$



The north-south distance between Reno and Los Angeles is about 670 km.

Example 3 FINDING A LENGTH USING $s = r\theta$

A rope is being wound around a drum with radius 0.8725 ft. How much rope will be wound around the drum if the drum is rotated through an angle of 39.72°?



The length of rope wound around the drum is the arc length for a circle of radius 0.8725 ft and a central angle of 39.72°.

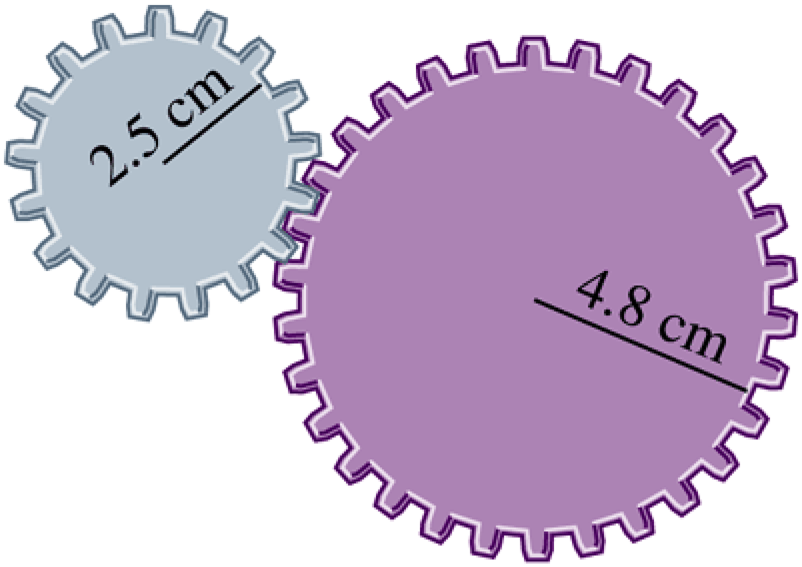
Example 3 FINDING A LENGTH USING $s = r\theta$ (continued)

Use $s = r\theta$, with the angle converted to radian measure.

$$s = r\theta = 0.8725\left[39.72\left(\frac{\pi}{180}\right)\right] \approx 0.6049 \text{ ft}$$

Example 4 FINDING AN ANGLE MEASURE USING $s = r\theta$

Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of 225°, through how many degrees will the larger gear rotate?

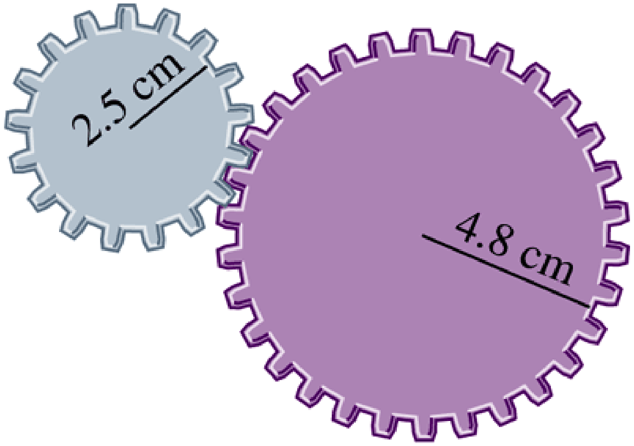


First find the radian measure of the angle of rotation for the smaller gear, and then find the arc length on the smaller gear. This arc length will correspond to the arc length of the motion of the larger gear.

Example 4 FINDING AN ANGLE MEASURE USING $s = r\theta$ (continued)

Since $225° = \frac{5\pi}{4}$ radians, for the smaller gear,

$$s = r\theta = 2.5\left(\frac{5\pi}{4}\right) = \frac{12.5\pi}{4} = \frac{25\pi}{8} \text{ cm}$$



An arc with this length on the larger gear corresponds to an angle measure θ:

$$s = r\theta$$

$$\frac{25\pi}{8} = 4.8\theta$$

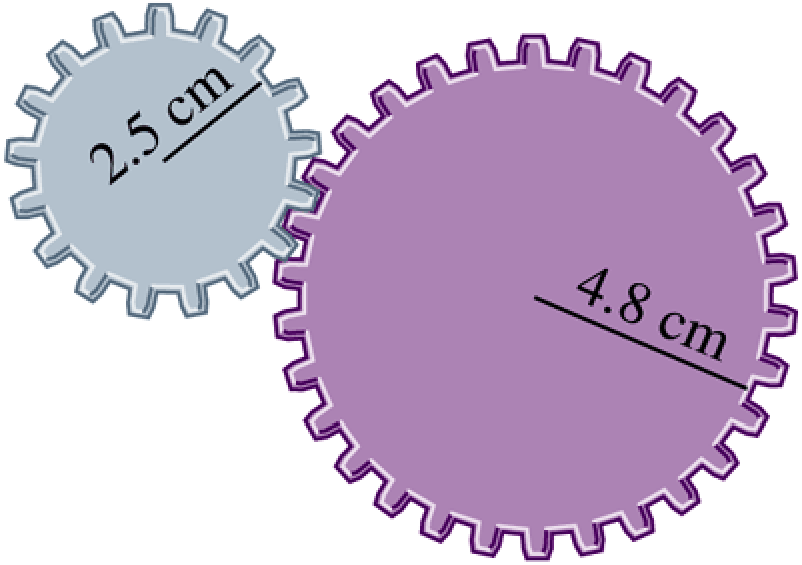
$$\frac{125\pi}{192} = \theta$$

Example 4 FINDING AN ANGLE MEASURE USING $s = r\theta$ (continued)

Convert θ to degrees:

$$\frac{125\pi}{192}\left(\frac{180^\circ}{\pi}\right) \approx 117^\circ$$

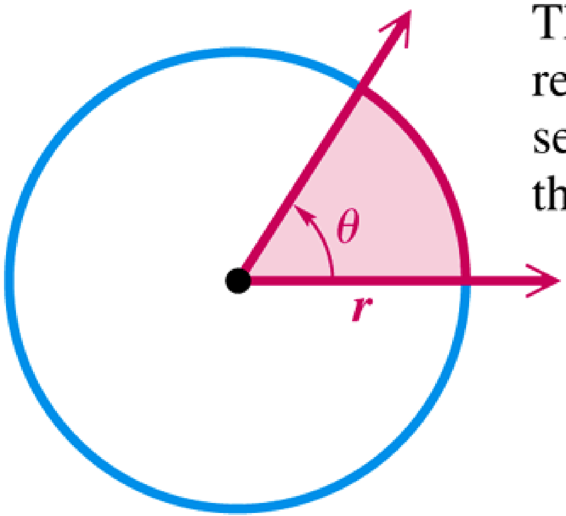
The larger gear rotates through an angle of 117°.



Area of a Sector of a Circle

A **sector of a circle** is the portion of the interior of a circle intercepted by a central angle.

Think of it as a "piece of pie."



The shaded region is a sector of the circle.

Area of a Sector

The area *A* of a sector of a circle of radius *r* and central angle *θ* is given by the following formula.

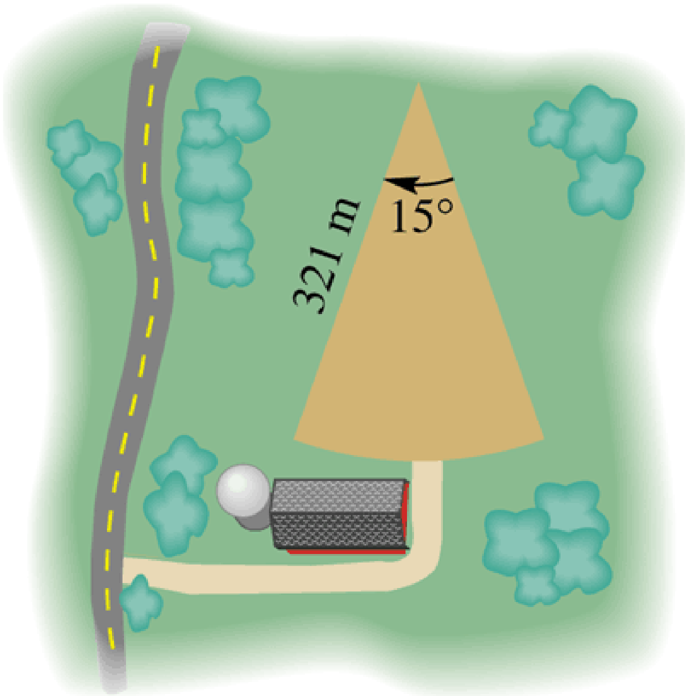
$$A = \frac{1}{2}r^2\theta, \quad \theta \text{ in radians}$$

Caution

As in the formula for arc length, the value of θ must be in radian mode when this formula is used for the area of a sector.

Example 5 FINDING THE AREA OF A SECTOR-SHAPED FIELD

A center-pivot irrigation system provides water to a sector-shaped field with the measures shown in the figure. Find the area of the field.

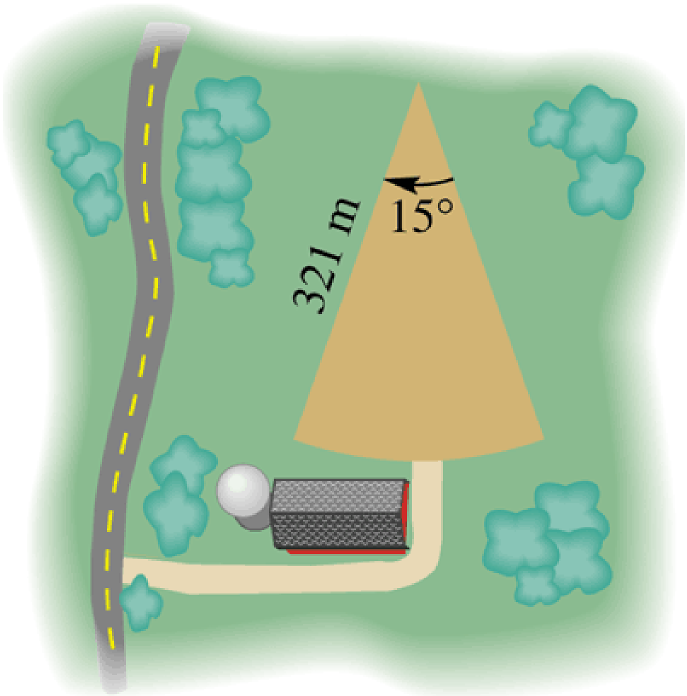


First, convert 15º to radians.

$$15° = 15\left(\frac{\pi}{180}\right) = \frac{\pi}{12} \text{ radian}$$

Example 7 FINDING THE AREA OF A SECTOR-SHAPED FIELD (continued)

Now use the formula to find the area of a sector of a circle with radius r = 321.



$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2}(321)^2 \left(\frac{\pi}{12}\right)$$

$$A \approx 13{,}500 \text{ m}^2$$