5.1 Fundamental Identities

Wednesday, October 25, 2017 9:16 AM

Goals:
* Fundamental Identities
* Use these fundamental identities to solve problems.

Reciprocal Identities

$$\boxed{\sec x = \frac{1}{\cos x} \; ; \; \csc x = \frac{1}{\sin x} \qquad \tan x = \frac{1}{\cot x} \; ; \; \cot x = \frac{1}{\tan x}}$$

Quotient Identities

$$\boxed{\tan x = \frac{\sin x}{\cos x} \; ; \; \cot x = \frac{\cos x}{\sin x}}$$

Pythagorean Identities

$$\boxed{\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \cot^2 x &= \csc^2 x \\ \tan^2 x + 1 &= \sec^2 x \end{aligned}}$$

Even - odd identities.

$$\boxed{\begin{aligned} &\sin(-x) = -\sin x \;;\; \cos(-x) = \cos x \\ &\tan(-x) = -\tan x \;;\; \cot(-x) = -\cot x \\ &\sec(-x) = \sec x \;;\; \csc(-x) = -\csc x \end{aligned}}$$

Application of these identities in solving problems.

E.g. $\tan x = -\frac{5}{3}$.

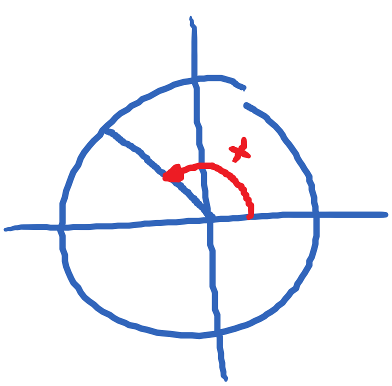
x is an angle in quadrant II.

Use one of the fundamental identities to find the given quantity.

(a) $\sec x = ?$

Identity: $\tan^2 x + 1 = \sec^2 x$

$$\left(-\frac{5}{3}\right)^2 + 1 = \sec^2 x$$



$$\frac{25}{9} + 1 = \sec^2 x$$

$$\frac{34}{9} = \sec^2 x$$

$\longrightarrow \sec x = \pm\sqrt{\frac{34}{9}}$. Since x is in quadrant II, $\sec x$ is negative.

So, $\sec x = -\sqrt{\frac{34}{9}} = \boxed{-\frac{\sqrt{34}}{3}}$

(b) $\sin x = ?$

$\sec x = \frac{1}{\cos x}$; $\cos x = \frac{1}{\sec x}$

So, $\cos x = -\frac{3}{\sqrt{34}}$

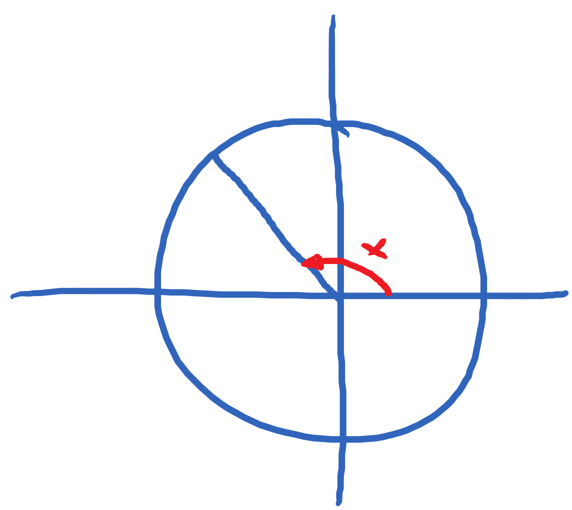
$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + \left(-\frac{3}{\sqrt{34}}\right)^2 = 1$

$\sin^2 x + \frac{9}{34} = 1$

$\sin^2 x = 1 - \frac{9}{34}$

$\sin^2 x = \frac{25}{34}$



$\sin x = \pm\sqrt{\frac{25}{34}}$. x is in II quadrant $\rightarrow$ sin x is positive.

is positive.

Thus, $\sin x = \sqrt{\dfrac{25}{34}} = \dfrac{5}{\sqrt{34}} \cdot \dfrac{\sqrt{34}}{\sqrt{34}} = \dfrac{5\sqrt{34}}{34}$

(c) $\cot(-x) = ?$

$$\cot(-x) = -\cot x$$
$$= -\frac{1}{\tan x}$$
$$= -\frac{1}{-\frac{5}{3}}$$

$$\boxed{\cot(-x) = \frac{3}{5}}$$

E.g. Write $\cos x$ in terms of $\tan x$

(Find an expression that only has $\tan x$ in it such that the expression equals to $\cos x$ for all angles x)

$\tan^2 x + 1 = \sec^2 x \rightarrow$ Pythagorean Identity

Reciprocal Identity: $\sec x = \dfrac{1}{\cos x}$.

$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$

Multiply both sides by $\cos^2 x$.

$$\cos^2 x \left(\tan^2 x + 1\right) = 1$$

$$\cos^2 x = \frac{1}{\tan^2 x + 1}$$

$$\cos x = \pm \sqrt{\frac{1}{\tan^2 x + 1}}$$

$$\cos x = \pm \frac{1}{\sqrt{\tan^2 x + 1}}.$$

E.g.

Write the expression $\dfrac{1+\cot^2 x}{1-\csc^2 x}$ in terms of $\sin x$ and $\cos x$ only and simplify

$$\frac{1+\cot^2 x}{1-\csc^2 x} = \frac{1+\left(\frac{\cos x}{\sin x}\right)^2}{1-\left(\frac{1}{\sin x}\right)^2}$$

$$= \frac{\frac{1\cdot\sin^2 x}{1\cdot\sin^2 x}+\frac{\cos^2 x}{\sin^2 x}}{\frac{1\cdot\sin^2 x}{1\cdot\sin^2 x}-\frac{1}{\sin^2 x}}$$

$$= \frac{\frac{\sin^2 x}{\sin^2 x}+\frac{\cos^2 x}{\sin^2 x}}{\frac{\sin^2 x}{\sin^2 x}-\frac{1}{\sin^2 x}}$$

$$= \frac{\dfrac{\sin^2 x + \cos^2 x}{\sin^2 x}}{\dfrac{\sin^2 x - 1}{\sin^2 x}} = \frac{\sin^2 x + \cos^2 x}{\cancel{\sin^2 x}} \cdot \frac{\cancel{\sin^2 x}}{\sin^2 x - 1}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x - 1} = \frac{1}{\sin^2 x - 1} = \frac{1}{-\cos^2 x}$$

$$= -\frac{1}{\cos^2 x}.$$