5.2. Verify Trig Identities

Monday, October 30, 2017 9:19 AM

$$\sec\theta = \frac{1}{\cos\theta} \quad ; \quad \csc\theta = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad ; \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

---

1 Verify Trig Identities by working from one side

$$\cot\theta + 1 = \csc\theta\left(\cos\theta + \sin\theta\right)$$

Verify that this identity is true for all angles θ.

Monday, October 30, 2017 9:28 AM

$$\underbrace{\cot\theta + 1}_{\text{LHS}} = \frac{\cos\theta}{\sin\theta} + 1 = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\sin\theta}$$

$$= \frac{\cos\theta + \sin\theta}{\sin\theta} = \frac{1}{\sin\theta}\cdot(\cos\theta + \sin\theta)$$

$$= \underbrace{\csc\theta(\cos\theta + \sin\theta)}_{\text{RHS}} \checkmark$$

This is going from LHS to RHS.

What if we want to start with RHS.

$$\underbrace{\csc\theta(\cos\theta + \sin\theta)}_{\text{RHS}} = \frac{1}{\sin\theta}(\cos\theta + \sin\theta)$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\sin\theta}$$

$$= \underbrace{\cot\theta + 1}_{\text{LHS}} \checkmark$$

Ex. Verify the identity

$$\tan^2\theta\left(1+\cot^2\theta\right) = \frac{1}{1-\sin^2\theta}$$

$$\text{LHS} = \tan^2\theta\left(1+\cot^2\theta\right)$$

$$= \frac{\sin^2\theta}{\cos^2\theta}\left(1+\cot^2\theta\right) \quad \text{(quotient identity)}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}\cdot \csc^2\theta \quad \text{(Pythagorean Identity)}$$

$$= \frac{\cancel{\sin^2\theta}}{\cos^2\theta}\cdot\frac{1}{\cancel{\sin^2\theta}} \quad \text{(Reciprocal Identity)}$$

$$= \frac{1}{\cos^2\theta} \quad \text{(Basic algebraic property of fractions)}$$

$$= \frac{1}{1-\sin^2\theta} = \text{RHS}. \quad \left(\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \cos^2\theta &= 1-\sin^2\theta\end{aligned}\right)$$

E.g. Verify the identity

$$\frac{\boxed{\tan(x) - \cot(x)}}{\boxed{\sin(x)\cos(x)}} = \sec^2 x - \csc^2 x.$$

$$RHS = \sec^2 x - \csc^2 x = \frac{1 \cdot \sin^2 x}{\cos^2 x \cdot \sin^2 x} - \frac{1 \cdot \cos^2 x}{\sin^2 x \cdot \cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x \cdot \sin^2 x} - \frac{\cos^2 x}{\sin^2 x \cdot \cos^2 x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cdot \cos^2 x}$$

$$= \frac{(\sin^2 x - \cos^2 x) / \sin x \cdot \cos x}{(\sin^2 x \cdot \cos^2 x) / \sin x \cdot \cos x}$$

$$= \frac{\boxed{\dfrac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x}}}{\boxed{\sin x \cdot \cos x}} = \frac{\dfrac{\sin^{\cancel{2}} x}{\cancel{\sin x} \cdot \cos x} - \dfrac{\cos^{\cancel{2}} x}{\sin x \cdot \cancel{\cos x}}}{\sin x \cdot \cos x}$$

$$= \frac{\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}}{\sin x \cdot \cos x} = \frac{\tan x - \cot x}{\sin x \cdot \cos x} = LHS.$$

2nd way: $$\frac{\tan x - \cot x}{\sin x \cdot \cos x} = \frac{\dfrac{\sin x \cdot \sin x}{\cos x \cdot \sin x} - \dfrac{\cos x \cdot \cos x}{\sin x \cdot \cos x}}{\sin x \cdot \cos x}$$

$$= \frac{\dfrac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x}}{\sin x \cdot \cos x} = \frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x} \cdot \frac{1}{\sin x \cdot \cos x}$$

$$= \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cdot \cos^2 x}$$

$$= \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x} \cdot \cos^2 x} - \frac{\cancel{\cos^2 x}}{\sin^2 \cdot \cancel{\cos^2 x}}$$

$$= \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \sec^2 x - \csc^2 x = RHS$$

E.g. Verify that: $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$.

Hint: Start with LHS, multiply both the top and the bottom by $(1 + \sin x)$. And go from there to get the RHS.

$$LHS = \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x + \cos x \cdot \sin x}{1 - \sin^2 x}$$

$$= \frac{\cancel{\cos x} \cdot (1 + \sin x)}{\cos^{\cancel{2}} x} = \frac{1 + \sin x}{\cos x} = RHS.$$

② Verify Identities by working with both sides

LHS = RHS

Common Expression.

E.x Verify the identity:

$$\frac{\sec x + \tan x}{\sec x - \tan x} = \frac{1 + 2\sin x + \sin^2 x}{\cos^2 x}$$

$$\text{LHS} = \frac{\frac{1}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\cos x} - \frac{\sin x}{\cos x}} = \frac{\frac{1+\sin x}{\cos x}}{\frac{1-\sin x}{\cos x}} = \frac{1+\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1-\sin x}$$

$$= \boxed{\frac{1+\sin x}{1-\sin x}}$$

$$\text{RHS} = \frac{1 + 2\sin x + \sin^2 x}{1 - \sin^2 x} = \frac{1 + \sin x + \sin x + \sin^2 x}{(1-\sin x)(1+\sin x)}$$

$$= \frac{(1+\sin x) + \sin x(1+\sin x)}{(1-\sin x)(1+\sin x)}$$

$$= \frac{(1+\sin x)\cancel{(1+\sin x)}}{(1-\sin x)\cancel{(1+\sin x)}} = \boxed{\frac{1+\sin x}{1-\sin x}}$$

$$(\text{Stuff } 1)^2 - (\text{Stuff } 2)^2$$

$$= (\text{Stuff } 1 - \text{Stuff } 2)(\text{Stuff } 1 + \text{Stuff } 2)$$

$\longrightarrow$ Difference between squares

$$A^2 \pm 2 \cdot AB + B^2 = (A \pm B)^2$$

(Square of a Sum / difference)