5.3-Sum and Difference Identities

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Obj 1: Sum and Difference Identity for Cosine

$45^\circ = \frac{\pi}{4}$; $30^\circ = \frac{\pi}{6}$; $75^\circ = \frac{5\pi}{12}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$; $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$; $\cos\left(\frac{5\pi}{12}\right) \stackrel{?}{=}$

$\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right) \neq \cos\left(\frac{5\pi}{12}\right)$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$ 0.2588

1.57

$\cos(45^\circ) + \cos(30^\circ) \neq \cos(75^\circ)$

$\cos(45^\circ) \cdot \cos(30^\circ) = 0.6123$

$\sin(45^\circ) \cdot \sin(30^\circ) = 0.3535$ −

0.2588

$\cos(75^\circ) = 0.2588$

Cosine of a Sum.

A and B are angles.

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

E.g. Find the exact value of the expression.

$$\cos 87^\circ \cos 93^\circ - \sin 87^\circ \sin 93^\circ = \cos(87^\circ + 93^\circ)$$

(87°: A, 93°: B, 87°: A, 93°: B)

Cosine of a sum identity

$$= \cos(180^\circ)$$

$$\cos 87^\circ \cos 93^\circ - \sin 87^\circ \sin 93^\circ = -1$$

E.g. Find the exact value of $\cos(165^\circ)$

$$\cos(165^\circ) = \cos(120^\circ + 45^\circ)$$

$$= \cos(120^\circ)\cdot\cos(45^\circ) - \sin(120^\circ)\cdot\sin(45^\circ)$$

$$= \left(-\frac{1}{2}\right)\cdot\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{-\sqrt{2}-\sqrt{6}}{4}$$

4 4 4

Cosine of a difference:

A, B : angles

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

E.g. Find the exact value of $\cos(15°)$

$$\cos 15° = \cos(60° - 45°) = \cos(60°)\cos(45°) + \sin(60°)\cdot\sin(45°)$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}.$$

$$\cos(15°) = \cos(45° - 30°) = \cos(45°)\cos(30°) + \sin(45°)\sin(30°)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$

Cosine of a Sum or Difference.

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$