Ex. Write $\cos(90° - \theta)$ as a trigonometric function of θ alone.

$$\cos(90° - \theta) = \cos(90°) \cdot \cos\theta + \sin(90°) \cdot \sin\theta$$
$$= \cancel{0 \cdot \cos\theta}^{\,0} + 1 \cdot \sin\theta$$
$$= \sin\theta$$

$$\cos(90° - \theta) = \sin\theta$$

Ex. Write the following as a trig function of θ alone

(a) $\sin(90° - \theta)$ (b) $\tan(90° - \theta)$

(c) $\sec(90° - \theta)$

(a) Since $\cos(90° - \text{Stuff}) = \sin(\text{Stuff})$,

$$\cos(90° - (90° - \theta)) = \sin(90° - \theta)$$
$$\cos(90° - 90° + \theta) = \sin(90° - \theta)$$
$$\cos\theta = \sin(90° - \theta)$$

So, $\boxed{\sin(90° - \theta) = \cos\theta}$

(b) $\tan(90° - \theta) = \dfrac{\sin(90° - \theta)}{\cos(90° - \theta)} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta.$

$$\tan(90° - \theta) = \cot\theta.$$

(c) $\sec(90° - \theta) = \dfrac{1}{\cos(90° - \theta)} = \dfrac{1}{\sin\theta} = \csc\theta$

$$\sec(90° - \theta) = \csc\theta.$$

So far, we have seen:

$$\boxed{\begin{aligned} &\cos(90° - \theta) = \sin\theta \;;\; \sin(90° - \theta) = \cos\theta \\ &\tan(90° - \theta) = \cot\theta \;;\; \cot(90° - \theta) = \tan\theta \\ &\sec(90° - \theta) = \csc\theta \;;\; \csc(90° - \theta) = \sec\theta \end{aligned}}$$

→ These are called cofunction identities.

E.g. Find a value of θ in the first quadrant so that $\cot\theta = \tan 25°$

By the cofunction identity, $\cot\theta = \tan(90° - \theta)$. So,

$$\tan(\boxed{90° - \theta}) = \tan(\boxed{25°})$$

Since both $90° - \theta$ and $25°$ are in the first quadrant, the only way for tangent to be equal is if the 2 angles are equal.

Hence, $90° - \theta = 25°$

$$\boxed{\theta = 65°}$$

E.g. $\cos A = -\frac{12}{13}$; $\sin B = \frac{3}{5}$.

A and B are angles in quadrant II.

Find $\cos(A+B)$.

$$\cos(A+B) = \boxed{\cos A}\cos B - \sin A\boxed{\sin B}$$

$$-\frac{12}{13} \qquad\qquad \frac{3}{5}$$

Given : $\cos A = -\frac{12}{13}$; A is in quadrant II.

Can you find $\sin A$?

$$\cos^2 A + \sin^2 A = 1$$

$$\left(-\frac{12}{13}\right)^2 + \sin^2 A = 1$$

$$\frac{144}{169} + \sin^2 A = 1$$

$$\sin^2 A = 1 - \frac{144}{169}$$

$$\sin^2 A = \frac{25}{169}$$

$$\sin A = \pm\sqrt{\frac{25}{169}}$$

Quadrant II, $\sin A > 0$

$$\sin A = \frac{5}{13}$$

Given $\sin B = \frac{3}{5}$, B is in the second quadrant,

find $\cos B$.

$$\cos^2 B + \sin^2 B = 1$$

$$\cos^2 B + \left(\frac{3}{5}\right)^2 = 1$$

$$\cos^2 B + \frac{9}{25} = 1$$

$$\cos^2 B = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos B = -\frac{4}{5} \quad (B \text{ is in quadrant II})$$

$$\cos(A+B) = -\frac{12}{13}\cdot\left(-\frac{4}{5}\right) - \frac{5}{13}\cdot\frac{3}{5}$$

$$= \frac{48}{65} - \frac{15}{65} = \boxed{\frac{33}{65}}$$