5.4. Sum and Difference Identities for Sine and Tangent

Monday, November 6, 2017 9:17 AM

Last time:

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\boxed{\cos(90^\circ - A) = \sin A}$$

$$\sin(90^\circ - A) = \cos A$$

Obj 1: Sum and Difference Identities for Sine

$$\sin(A+B) = \cos(90^\circ - (A+B))$$

$$= \cos\left[\boxed{90^\circ - A - B}\right]$$

Difference Identity for Cosine

$$= \cos\left[(90^\circ - A) - B\right]$$

$$= \cos(90^\circ - A) \cdot \cos B + \sin(90^\circ - A) \cdot \sin B$$

$$= \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\boxed{\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B}$$

What about $\sin(A-B)$?

$$\begin{aligned}
\sin(A-B) &= \sin(A+(-B)) \\
&= \sin A \cdot \cos(-B) + \cos A \cdot \sin(-B) \\
&= \sin A \cdot \cos B + \cos A \cdot (-\sin B) \\
&= \sin A \cdot \cos B - \cos A \sin B.
\end{aligned}$$

$$\boxed{\sin(A-B) = \sin A \cos B - \cos A \sin B}$$

Sine of a Sum or Difference.

$$\boxed{\begin{aligned}
\sin(A+B) &= \sin A \cos B + \cos A \sin B \\
\sin(A-B) &= \sin A \cos B - \cos A \sin B
\end{aligned}}$$

Ex. 1 Find the exact value of $\sin 75°$

$$\sin 75° = \sin(45° + 30°)$$
$$= \sin(45°)\cos(30°) + \cos(45°)\sin(30°)$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$\sin 75° = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Ex. 2. Find the exact value of $\sin 40° \cos 160° - \cos 40° \sin 160°$.

$$\sin 40° \cos 160° - \cos 40° \sin 160° = \sin(40° - 160°)$$
$$= \sin(-120°)$$
$$= -\sin(120°)$$
$$= \boxed{-\frac{\sqrt{3}}{2}}$$

Ex. 3. Write the following function as an expression that involves trig functions of θ only

$$\sin(180° - \theta) = \sin(180°)\cdot\cos\theta - \cos(180°)\cdot\sin\theta$$

$$= \cancel{0\cdot\cos\theta}^{\,0} - (-1)\cdot\sin\theta$$

$$\sin(180° - \theta) = \sin\theta$$

Ex. 4. $\sin A = \frac{4}{5}$; $\frac{\pi}{2} < A < \pi$ (Quadrant II)

$\cos B = -\frac{5}{13}$; $\pi < B < \frac{3\pi}{2}$ (Quadrant III)

Find $\sin(A+B)$.

Sol: $\sin(A+B) = \boxed{\sin A}\boxed{\cos B} + \boxed{\cos A}\cdot\boxed{\sin B}$

? ?

$\frac{4}{5}\cdot\left(-\frac{5}{13}\right) \quad \left(-\frac{3}{5}\right)\cdot\left(-\frac{12}{13}\right) \to$ Simplify.

$\sin^2 A + \cos^2 A = 1$ $\left(\frac{4}{5}\right)^2 + \cos^2 A = 1$	$\frac{16}{25} + \cos^2 A = 1$ $\cos^2 A = 1 - \frac{16}{25} = \frac{9}{25}$

$\cos A = -\frac{3}{5}$ (A is in II quadrant)

$$\sin^2 B + \cos^2 B = 1$$

$$\sin^2 B + \left(-\frac{5}{13}\right)^2 = 1$$

$$\sin^2 B + \frac{25}{169} = 1$$

$$\sin^2 B = 1 - \frac{25}{169}$$

$$\sin^2 B = \frac{144}{169} \longrightarrow \sin B = -\frac{12}{13}$$

(B is in III quadrant)

Ex. 5. Verify the identity.

$$\boxed{\sin\left(\frac{\pi}{6}+\theta\right) + \cos\left(\frac{\pi}{3}+\theta\right) = \cos\theta}$$

$$\text{LHS} = \sin\left(\frac{\pi}{6}+\theta\right) + \cos\left(\frac{\pi}{3}+\theta\right)$$

$$= \sin\left(\frac{\pi}{6}\right)\cos\theta + \cos\left(\frac{\pi}{6}\right)\sin\theta + \cos\left(\frac{\pi}{3}\right)\cdot\cos\theta - \sin\left(\frac{\pi}{3}\right)\cdot\sin\theta$$

$$= \frac{1}{2}\cos\theta + \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{1}{2}\cos\theta \cancel{- \frac{\sqrt{3}}{2}\sin\theta}$$

$$= \cos\theta = \text{RHS}$$