## Obj 2: Sum and Difference Identities for Tangent.

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{(\sin A \cos B + \cos A \sin B)/\cos A \cdot \cos B}{(\cos A \cos B - \sin A \sin B)/\cos A \cdot \cos B}$$

$$= \frac{\dfrac{\sin A \cancel{\cos B}}{\cos A \cdot \cancel{\cos B}} + \dfrac{\cancel{\cos A} \cdot \sin B}{\cancel{\cos A} \cdot \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{1 - \dfrac{\sin A}{\cos A} \cdot \dfrac{\sin B}{\cos B}}$$

$$\boxed{\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}}$$

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}}$$

Ex. 6. Find the exact value of $\tan\frac{7\pi}{12}$.

$$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\cdot\tan\frac{\pi}{4}}$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}\cdot 1} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{\sqrt{3} + 3 + 1 + \sqrt{3}}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2}$$

$$\tan\left(\frac{7\pi}{12}\right) = -2 - \sqrt{3}$$

Ex. 7.

$\sin A = \frac{4}{5}$; $\frac{\pi}{2} < A < \pi$

$\cos B = -\frac{5}{13}$; $\pi < B < \frac{3\pi}{2}$.

Find $\tan(A+B)$

$$\tan(A+B) = \frac{\overset{-\frac{4}{3}}{\tan A} + \tan B}{1 - \tan A \tan B}$$

$$\tan A = \frac{\overset{\frac{4}{5}}{\sin A}}{\underset{-\frac{3}{5}}{\cos A}}\ ;\quad \tan A = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{\cancel{5}} \cdot \frac{\cancel{5}}{3} = -\frac{4}{3}$$

$\tan A = -\frac{4}{3}$.

$$\tan B = \frac{\sin B}{\cos B} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{\cancel{13}} \cdot \frac{\cancel{13}}{5} = \frac{12}{5}.$$

So, $\tan B = \frac{12}{5}$.

$$\frac{-\frac{4}{3} + \frac{12}{5}}{1 - \left(-\frac{4}{3}\right) \cdot \frac{12}{5}} = \dots$$

Simplify

Ex. 8 Write the following function as trig functions of $\theta$ only.

$\tan(45^\circ - \theta)$

$$\tan(45^\circ - \theta) = \frac{\tan 45^\circ - \tan\theta}{1 + \tan 45^\circ \cdot \tan\theta}$$

$$\tan(45^\circ - \theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$$

---

Ex. 9 Verify the identity:

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}.$$

$$LHS = \tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \cdot \tan A}$$

$$= \frac{2\tan A}{1 - \tan^2 A} = RHS$$