5.5. Double Angle Identities and Product-to-Sum and Sum-to-Product Identities.

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Last time:

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B.$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

Obj 1: Double Angle Identities.

$$\cos(2A) = \cos(A+A) = \cos A \cdot \cos A - \sin A \cdot \sin A$$

$$= \cos^2 A - \sin^2 A$$

$$\boxed{\cos(2A) = \cos^2 A - \sin^2 A}$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$= (1 - \sin^2 A) - \sin^2 A$$

$$= 1 - \sin^2 A - \sin^2 A$$

$$\boxed{\cos(2A) = 1 - 2\sin^2 A}$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

$$= \cos^2 A - (1 - \cos^2 A)$$

$$= \cos^2 A - 1 + \cos^2 A$$

$$\boxed{\cos(2A) = 2\cos^2 A - 1}$$

Double-angle identity for sine

$$\sin(2A) = \sin(A + A) = \sin A \cos A + \cos A \sin A$$

$$= \sin A \cos A + \sin A \cos A$$

$$\boxed{\sin(2A) = 2\sin A \cos A}$$

Double-Identity for tangent

$$\tan(2A) = \tan(A+A) = \frac{\tan A + \tan A}{1 - \tan A \cdot \tan A}$$

$$\boxed{\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}}$$

All the Double-Angle Identities:

$$\cos(2A) = \cos^2 A - \sin^2 A \;;\quad \cos(2A) = 2\cos^2 A - 1$$

$$\cos(2A) = 1 - 2\sin^2 A$$

$$\sin(2A) = 2\sin A \cos A$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

Ex. 1. Given that $\cos\theta = \frac{3}{5}$ and $\sin\theta < 0$

Q: Find $\cos(2\theta)$, $\sin(2\theta)$ and $\tan(2\theta)$

From one of the double-angle identities for cosine

$$\cos(2\theta) = 2\cos^2\theta - 1$$

$$= 2\cdot\left(\frac{9}{25}\right) - 1 = \frac{18}{25} - 1 = \boxed{-\frac{7}{25}}$$

So, $\boxed{\cos(2\theta) = -\frac{7}{25}}$.

Find $\sin\theta$. $\quad \sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta + \frac{9}{25} = 1$$

$$\sin^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$$

Since $\sin\theta < 0$, we must have $\sin\theta = -\frac{4}{5}$.

$$\sin(2\theta) = 2\sin\theta\cdot\cos\theta = 2\cdot\left(-\frac{4}{5}\right)\cdot\left(\frac{3}{5}\right)$$

$$\boxed{\sin(2\theta) = -\frac{24}{25}}$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{-\frac{24}{25}}{-\frac{7}{25}} = \frac{24}{25}\cdot\frac{25}{7} = \frac{24}{7}.$$

$$\boxed{\tan(2\theta) = \frac{24}{7}}$$