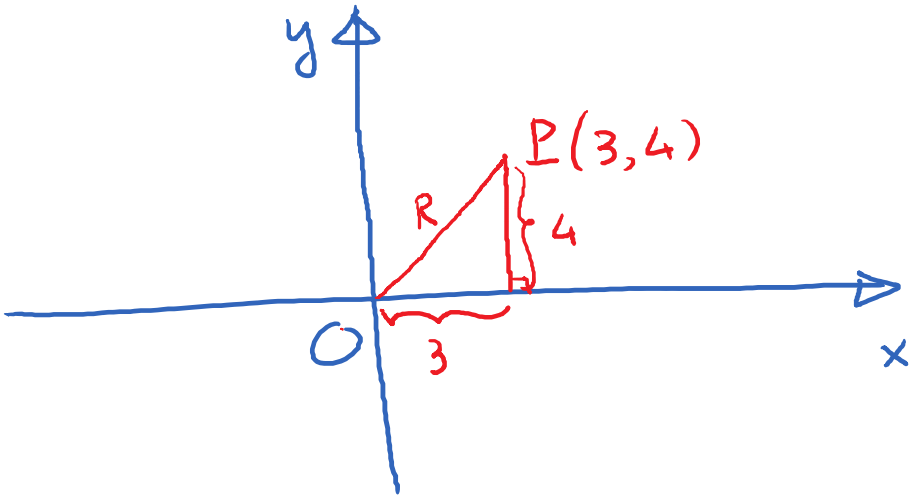
1.3. Trigonometric Functions

Wednesday, September 13, 2017 9:15 AM

Objective: Define 6 basic trig functions.

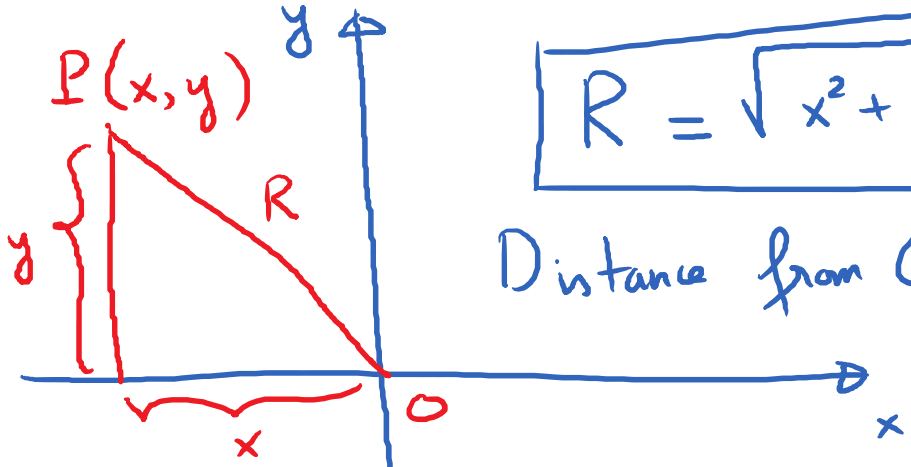
Distance formula



Distance from O to P:

$$R^2 = 3^2 + 4^2$$

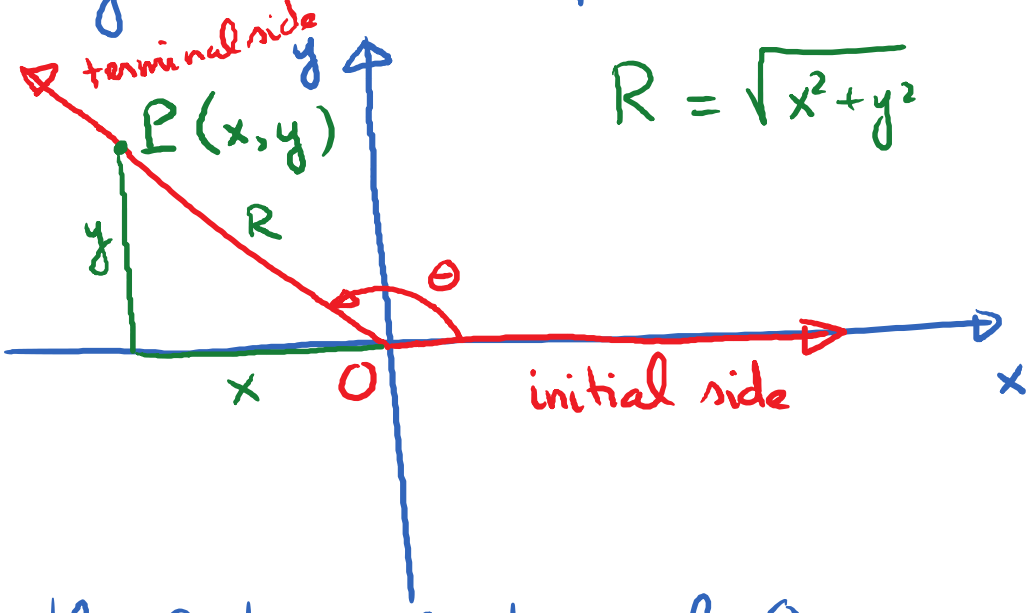
$$R = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$



$$R = \sqrt{x^2 + y^2}$$

Distance from O to P(x,y)

θ : any angle in standard position.



$$R = \sqrt{x^2+y^2}$$

Define the 6 trig functions of θ.

$\sin\theta$
(read as sine of theta)

$$\sin\theta = \frac{y}{R}$$

$\cos\theta$
(read as cosine of theta)

$$\cos\theta = \frac{x}{R}$$

$\csc\theta$

(read as cosecant of θ)

$$\csc\theta = \frac{R}{y} \quad (y \neq 0)$$

$\sec\theta$

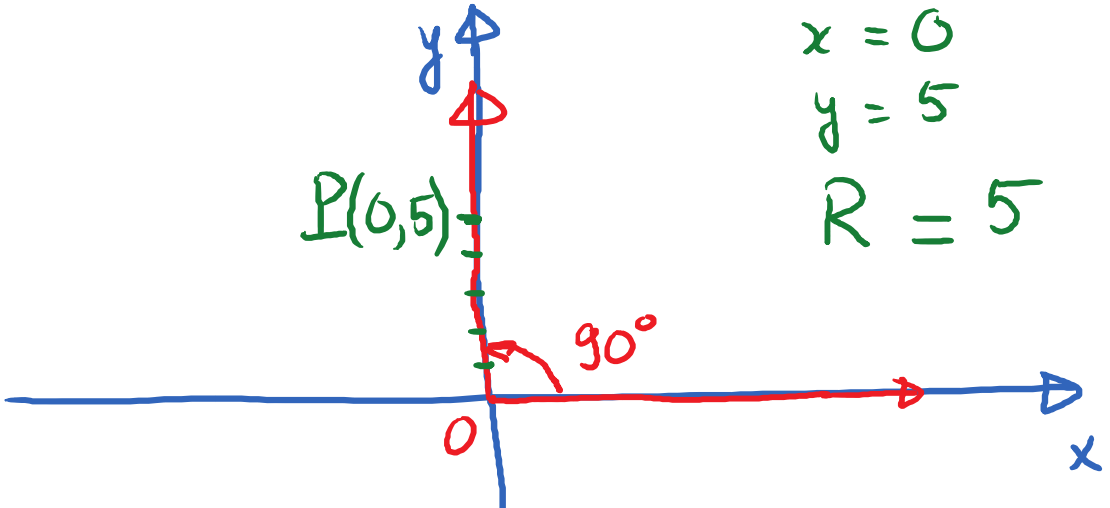
(read as secant of θ)

$$\sec\theta = \frac{R}{x} \quad (x \neq 0)$$

$$\tan\theta = \frac{y}{x} \quad (x \neq 0) \; ; \quad \cot\theta = \frac{x}{y} \quad (y \neq 0)$$

(tangent of theta) (cotangent of theta)

E.g. Calculate 6 trig functions for $\theta = 90°$.



$x = 0$

$y = 5$

$R = 5$

$$\sin(90°) = \frac{y}{R} = \frac{5}{5} = 1 . \quad \sin(90°) = 1$$

$$\cos(90^\circ) = \frac{x}{R} = \frac{0}{5} = 0$$

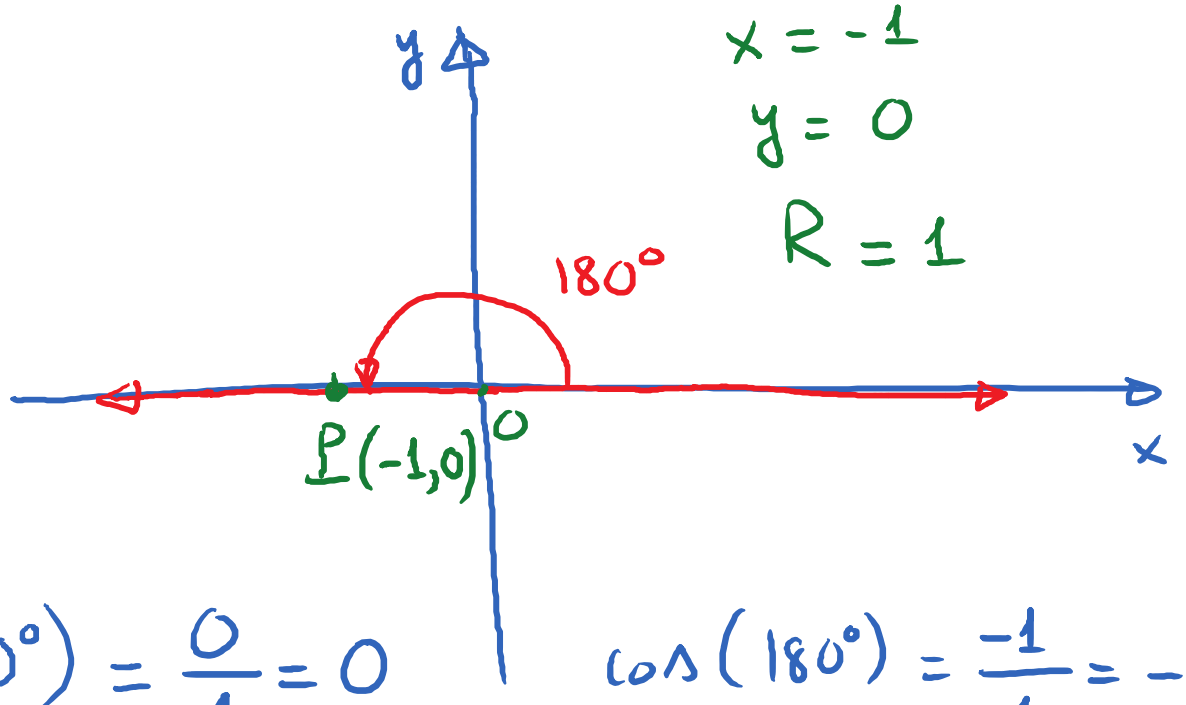
$$\csc(90^\circ) = \frac{R}{y} = \frac{5}{5} = 1.$$

$$\sec(90^\circ) = \frac{R}{x} = \frac{5}{0} = \text{undefined}.$$

$$\tan(90^\circ) = \frac{y}{x} = \frac{5}{0} = \text{undefined}.$$

$$\cot(90^\circ) = \frac{x}{y} = \frac{0}{5} = 0$$

E.g.



$x = -1$

$y = 0$

$R = 1$

$$\sin(180^\circ) = \frac{0}{1} = 0$$

$$\cos(180^\circ) = \frac{-1}{1} = -1$$

$$\csc(180°) = \frac{1}{0} = \text{undefined}$$

$$\sec(180°) = \frac{1}{-1} = -1$$

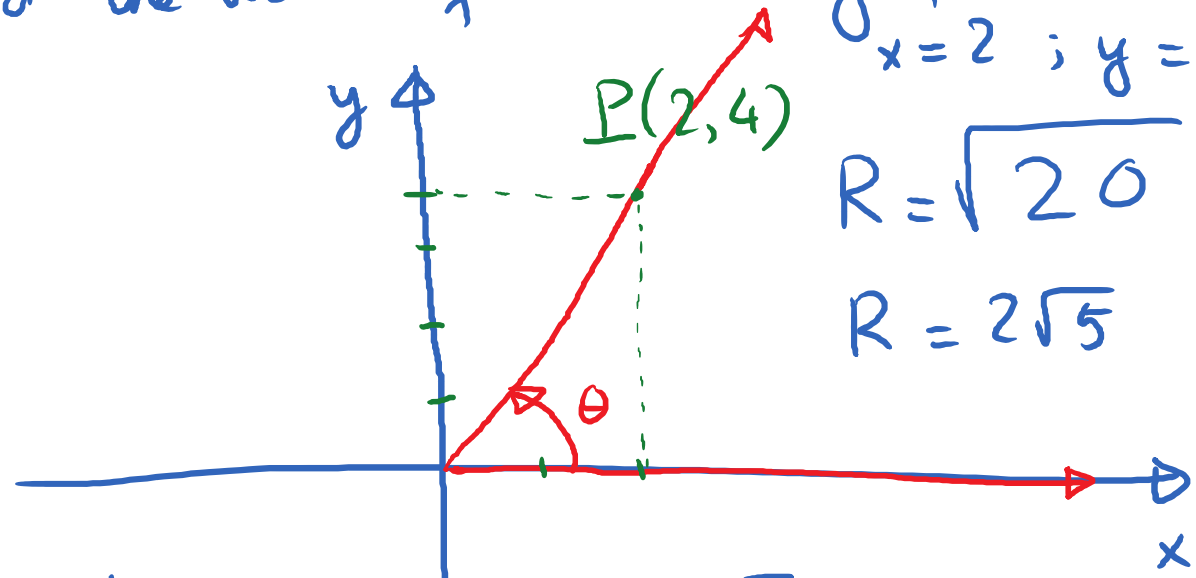
$$\tan(180°) = \frac{0}{-1} = 0$$

$$\cot(180°) = \frac{-1}{0} = \text{undefined}$$

E.g. The terminal side of an angle θ in standard position passes through the point $(2,4)$.

Find the values of the 6 trig functions of θ.

$x = 2$; $y = 4$



$$R = \sqrt{20}$$

$$R = 2\sqrt{5}$$

$$\sin\theta = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos\theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\csc\theta = \frac{5}{2\sqrt{5}} = \frac{5\sqrt{5}}{2\sqrt{5}\cdot\sqrt{5}} = \frac{5\sqrt{5}}{10} = \frac{\sqrt{5}}{2}.$$

$$\sec\theta = \frac{5}{\sqrt{5}} = \sqrt{5}$$

$$\tan\theta = \frac{4}{2} = 2$$

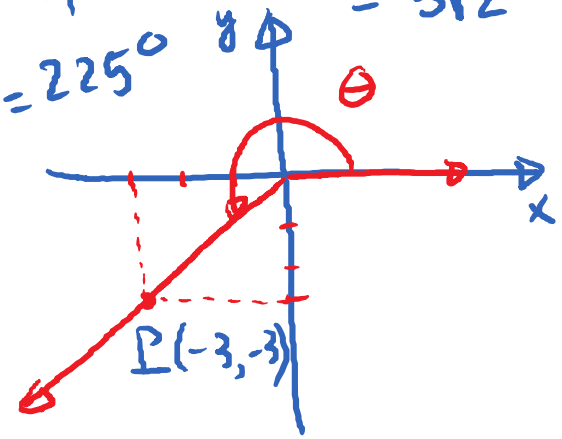
$$\cot\theta = \frac{1}{2}.$$

E.g. The terminal side of an angle θ in standard position passes through the point $(-3,-3)$.

Q: Find the values of all the trig functions of θ.

$R = \sqrt{18} = 3\sqrt{2}$

$\theta = 225°$



$$\sin\theta = \frac{y}{R} = \frac{-3}{3\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

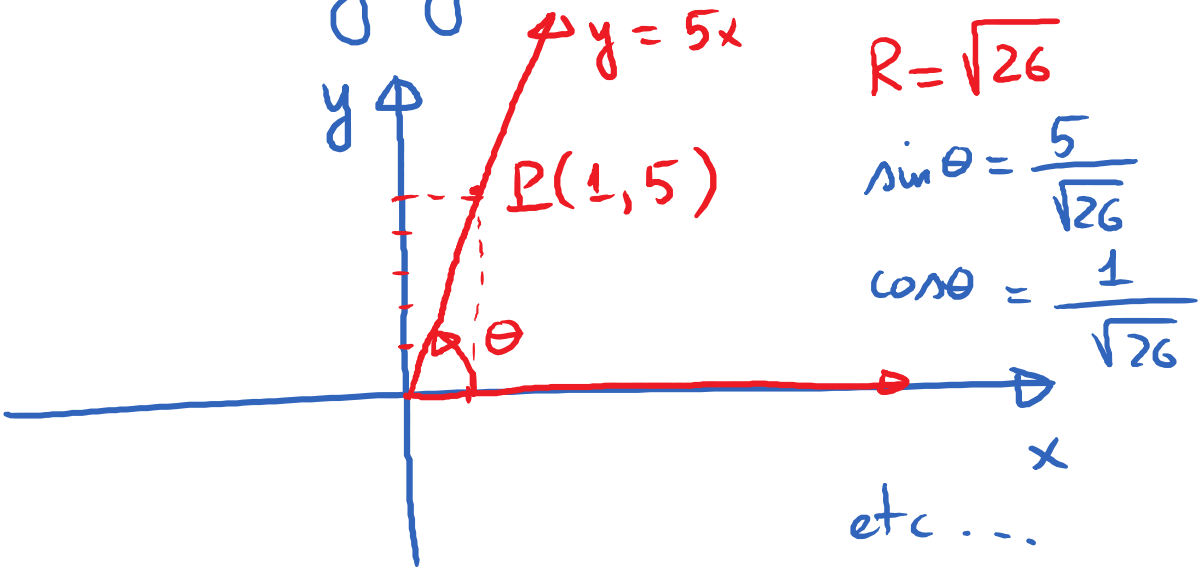
$$\cos\theta = \frac{x}{R} = \frac{-3}{3\sqrt{2}} = \frac{-\sqrt{2}}{2}.$$

$$\csc\theta = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$$

$\sec\theta = -\sqrt{2}$. $\tan\theta = 1$

$\cot\theta = 1$

E.g. Find the values of the 6 trig functions of the angle θ in standard position whose terminal side is the ray $y = 5x$, $x \geq 0$



$R = \sqrt{26}$

$\sin\theta = \frac{5}{\sqrt{26}}$

$\cos\theta = \frac{1}{\sqrt{26}}$

etc ...

θ	$\sin\theta$	$\cos\theta$	$\csc\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
90°						
180°						
270°						
360°						
0°						

Fill out this example