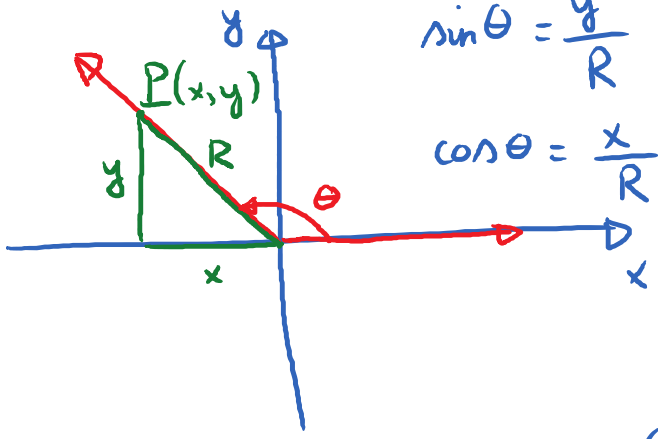
1.4. Using the definitions of the trig functions

Wednesday, September 13, 2017 10:21 AM

Objective 1: Reciprocal Identities.



$\sin\theta = \frac{y}{R}$; $\csc\theta = \frac{R}{y}$

$\cos\theta = \frac{x}{R}$; $\sec\theta = \frac{R}{x}$

$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

$$\sin\theta = \frac{1}{\csc\theta} \quad ; \quad \csc\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad ; \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad ; \quad \cot\theta = \frac{1}{\tan\theta}$$

E.g. ① $\cos\theta$ given that $\sec\theta = \frac{5}{3}$

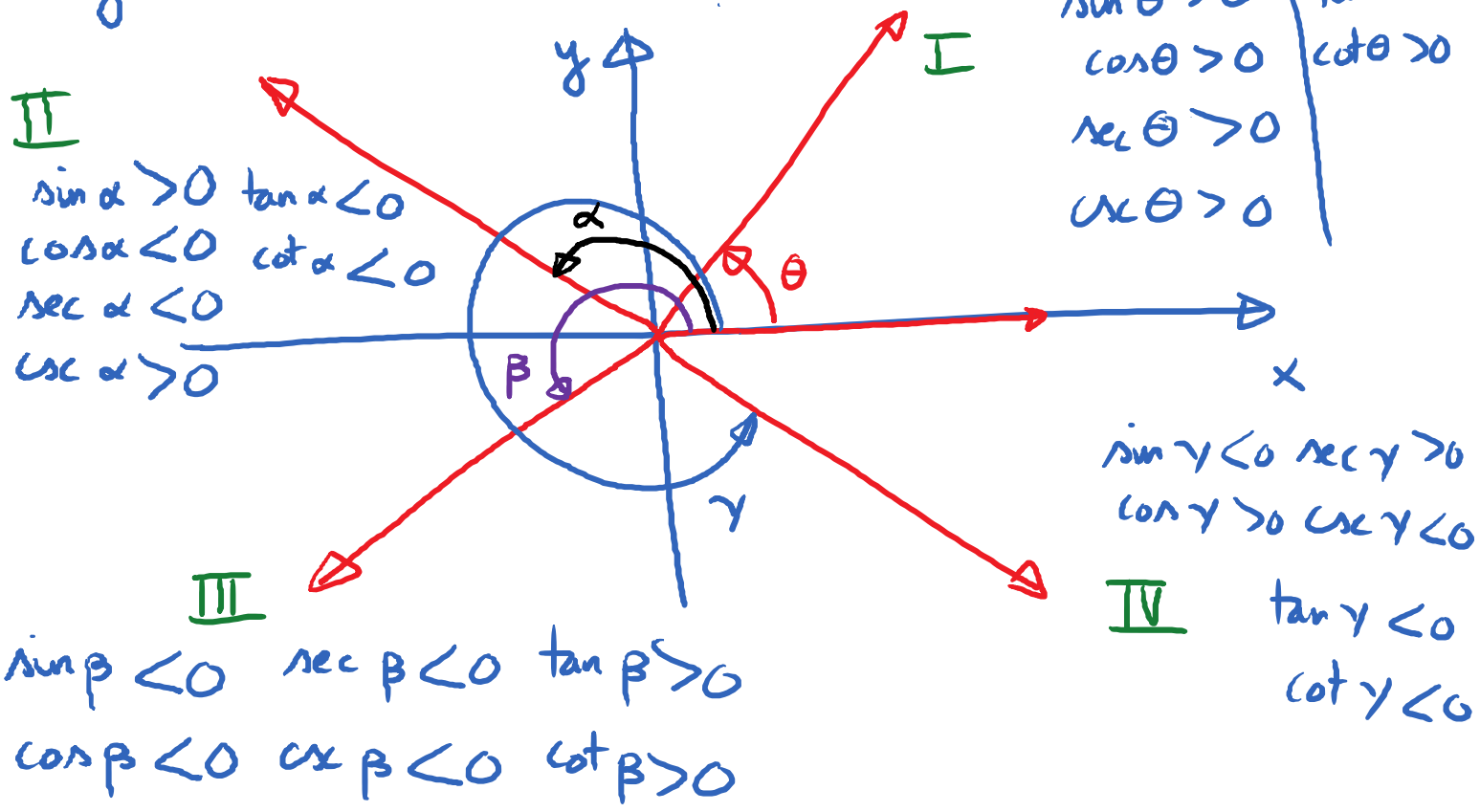
② Find $\sin\theta$ given that $\csc\theta = -\frac{\sqrt{12}}{3}$

Sol.

① $= \frac{3}{5}$

② $\sin\theta = -\frac{\sqrt{3}}{2}$.

Signs of Function Values.



I: $\sin\theta > 0$, $\cos\theta > 0$, $\sec\theta > 0$, $\csc\theta > 0$ | $\tan\theta > 0$, $\cot\theta > 0$

II: $\sin\alpha > 0$, $\cos\alpha < 0$, $\sec\alpha < 0$, $\csc\alpha > 0$ | $\tan\alpha < 0$, $\cot\alpha < 0$

III: $\sin\beta < 0$, $\cos\beta < 0$ | $\sec\beta < 0$, $\csc\beta < 0$ | $\tan\beta > 0$, $\cot\beta > 0$

IV: $\sin\gamma < 0$, $\cos\gamma > 0$ | $\sec\gamma > 0$, $\csc\gamma < 0$ | $\tan\gamma < 0$, $\cot\gamma < 0$

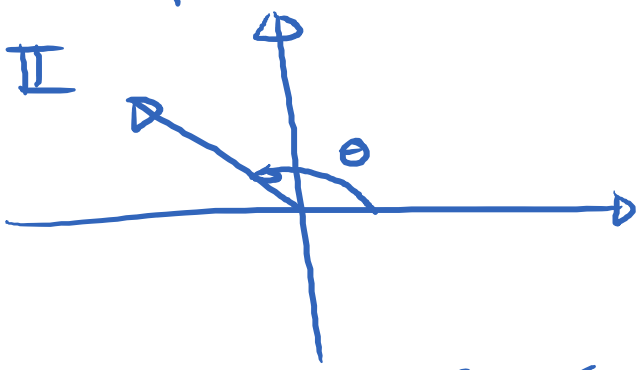
E.g. $\cos(97^\circ) < 0$

$\tan(375^\circ) > 0$

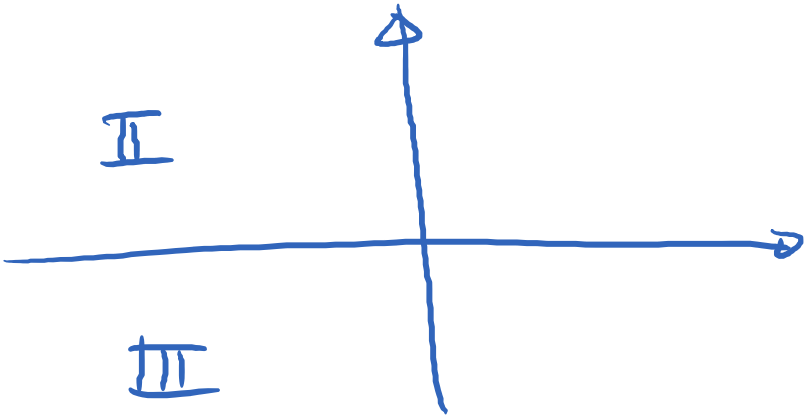
E.g. θ is an angle in standard position.

(a) $\sin\theta > 0$, $\tan\theta < 0$

Which quadrant does θ belong to?



(b) $\cos\theta < 0$, $\sec\theta < 0$



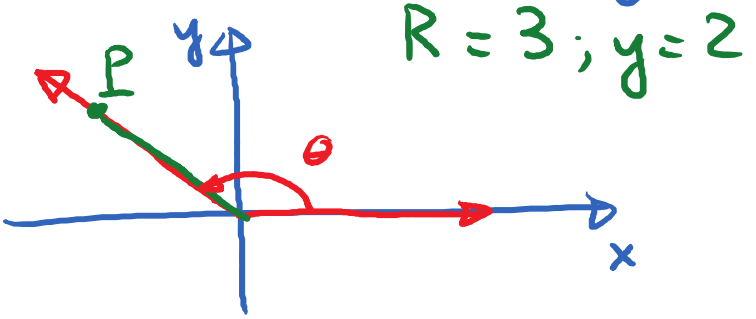
Range of Trig functions

Function of θ	Range	Range in interval notation
$\sin\theta$	$-1 \leq \sin\theta \leq 1$	$[-1, 1]$
$\cos\theta$	$-1 \leq \cos\theta \leq 1$	$[-1, 1]$
$\sec\theta$	$\sec\theta \geq 1$ or $\sec\theta \leq -1$	$(-\infty, -1] \cup [1, \infty)$
$\csc\theta$	$\csc\theta \geq 1$ or $\csc\theta \leq -1$	$(-\infty, -1] \cup [1, \infty)$
$\tan\theta$		$(-\infty, \infty)$
$\cot\theta$		$(-\infty, \infty)$

E.g. Given θ is an angle in quadrant II and

$$\sin\theta = \frac{2}{3}.$$

Q: Find the values of the other 5 trig functions of θ



Sol: $\csc\theta = \frac{3}{2}$.

$R^2 = x^2 + y^2$; $9 = x^2 + 4$; $x^2 = 5$

$x = \pm\sqrt{5}$ ⟶ Since θ is in quadrant II, $x = -\sqrt{5}$

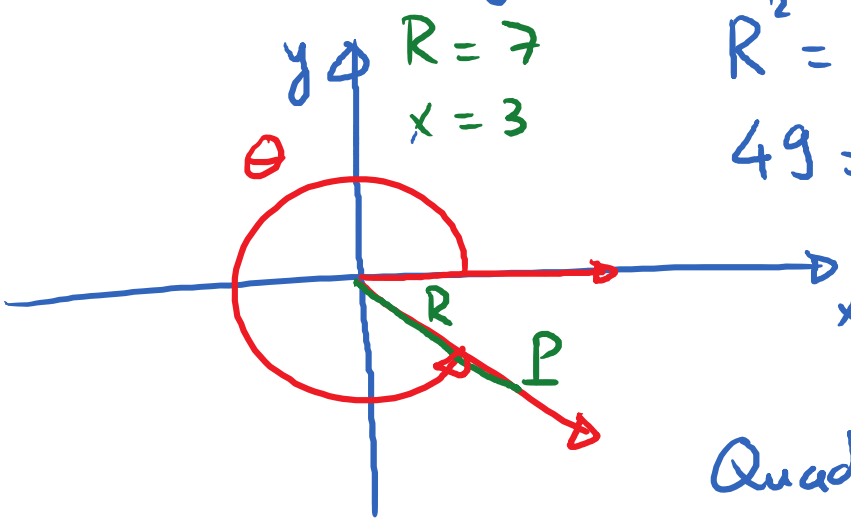
$x = -\sqrt{5}$; $y = 2$; $R = 3$

⟶ piece of cake to find remaining trig functions.

E.g. θ to be in quadrant IV

$\sec\theta = \frac{7}{3}$.

Find the remaining trig functions of θ.



$R^2 = x^2 + y^2$

$49 = 9 + y^2$

$y^2 = 40$

$y = \pm\sqrt{40}$

Quadrant IV:

$y = -\sqrt{40}$