$$\sin\theta = \frac{y}{R} = \frac{-\sqrt{40}}{7} = \frac{-2\sqrt{10}}{7}$$

$$\cos\theta = \frac{3}{7}$$

$$\csc\theta = \frac{R}{y} = -\frac{7}{\sqrt{40}} \cdot \frac{\sqrt{40}}{\sqrt{40}} = -\frac{7\sqrt{40}}{40}$$

$$\tan\theta = \frac{-\sqrt{40}}{3} = \frac{-2\sqrt{10}}{3}$$

$$\cot\theta = -\frac{3}{\sqrt{40}} = -\frac{3\sqrt{40}}{40} = -\frac{6\sqrt{10}}{40}$$

$$= -\frac{3\sqrt{10}}{20}$$

Pythagorean Identity

(1) $(\sin\theta)^2 + (\cos\theta)^2 = 1$; true

<u>Notation:</u> $\boxed{\sin^2\theta + \cos^2\theta = 1}$ for any angle θ

2) $(\tan\theta)^2 + 1 = (\sec\theta)^2$, for any angle θ

Notation: $\tan^2\theta + 1 = \sec^2\theta$

(Reason why this is true:

$$LHS = \left(\frac{y}{x}\right)^2 + 1 = \frac{y^2}{x^2} + 1 = \frac{y^2}{x^2} + \frac{x^2}{x^2}$$

$$= \frac{y^2+x^2}{x^2} = \frac{R^2}{x^2} = \left(\frac{R}{x}\right)^2$$

$$= (\sec\theta)^2 = RHS$$

3) $\cot^2\theta + 1 = \csc^2\theta$, true for all angles θ for which $\cot\theta$ & $\csc\theta$ are defined.

Pythagorean Identity:

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\tan^2\theta + 1 = \sec^2\theta$
3. $\cot^2\theta + 1 = \csc^2\theta$

Quotient Identities:

1. $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$ (provided that $\cos\theta \neq 0$)

2. $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$ (provided that $\sin\theta \neq 0$)

E.g. Given that $\cos\theta = -\dfrac{\sqrt{3}}{4}$ and $\boxed{\sin\theta > 0}$

Q: Use Identities to find $\sin\theta$ and $\tan\theta$

Sol:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1$$

$$\sin^2\theta + \frac{3}{16} = 1$$

$$\sin^2\theta = 1 - \frac{3}{16}$$

$$\sin^2\theta = \frac{13}{16}$$

$$\boxed{-\frac{\sqrt{39}}{3} = \tan\theta}$$

$$\sin\theta = \pm\frac{\sqrt{13}}{4}$$

Since we are given that $\sin\theta > 0$, we must have

$$\sin\theta = \frac{\sqrt{3}}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{13}}{4}}{-\frac{\sqrt{3}}{4}} = -\frac{\sqrt{13}}{4}\cdot\frac{4}{\sqrt{3}} = \frac{-\sqrt{13}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

E.g. Given that $\tan\theta = \frac{4}{3}$ and θ is in quadrant III. Find $\sin\theta$ and $\cos\theta$ using identities.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{3}{5}\right)^2 = 1$$

$$\sin^2\theta + \frac{9}{25} = 1$$

$$\sin^2\theta = \frac{16}{25}$$

$$\sin\theta = \pm\frac{4}{5}$$

Since θ is in quadrant III, $\sin\theta = -\frac{4}{5}$.

2nd way to find $\sin\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{(Quotient Identity)}$$

$$\frac{4}{3} = \frac{\sin\theta}{-\frac{3}{5}} \longrightarrow \sin\theta = \frac{4}{\cancel{3}}\cdot\left(-\frac{\cancel{3}}{5}\right)$$

$$\sin\theta = -\frac{4}{5}$$