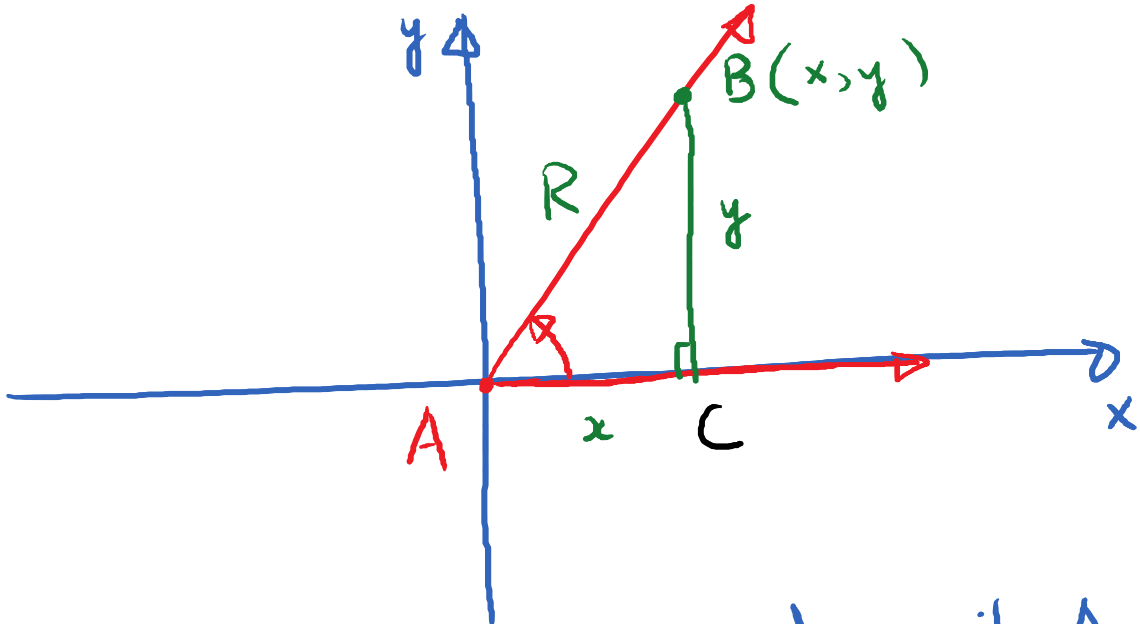
2.1. Trigonometric functions of Acute Angles.

Wednesday, September 20, 2017 9:27 AM

Obj. #1: Right-triangle definitions of trig functions

Consider an acute angle



$$\sin A = \frac{y}{R} = \frac{BC}{AB} = \frac{\text{side opposite } A}{\text{hypotenuse}}$$

$$\cos A = \frac{x}{R} = \frac{AC}{AB} = \frac{\text{side adjacent to } A}{\text{hypotenuse}}$$

$$\tan A = \frac{y}{x} = \frac{BC}{AC} = \frac{\text{side opposite } A}{\text{side adjacent to } A}.$$

$\csc A$; $\sec A$, $\cot A$

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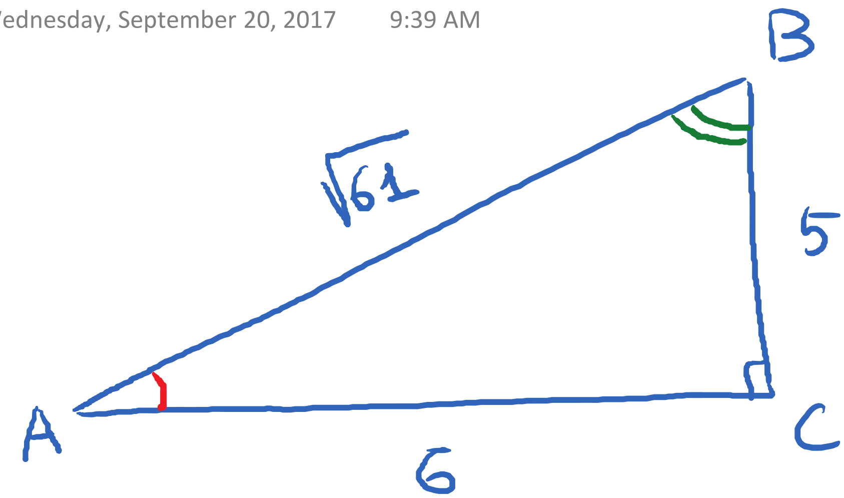
E.g.



$AB = \sqrt{61}$

Find $\sin A$, $\cos A$, $\tan A$, $\sec A$, $\csc A$, $\cot A$

Find $\sin B$, $\cos B$, $\tan B$, $\sec B$, $\csc B$, $\cot B$.

$\sin A = \frac{5}{\sqrt{61}}$; $\cos A = \frac{6}{\sqrt{61}}$, $\tan A = \frac{5}{6}$, $\sec A = \frac{\sqrt{61}}{6}$, ...

$\sin B = \frac{6}{\sqrt{61}}$, $\cos B = \frac{5}{\sqrt{61}}$, $\cot B = \frac{5}{6}$, $\csc B = \frac{\sqrt{61}}{6}$

$\sin A = \cos B$, $\cos A = \sin B$; $\cot A = \tan B$

$\tan A = \cot B$, $\sec A = \csc B$; $\csc A = \sec B$

A and B are complementary angles ($A + B = 90°$)

If 2 angles are complementary; i.e., the sum of their measurements equals 90°, then the sine of one angle equals the cosine of the other. The tangent of one angle equals the cotangent of the other, the secant of one angle equals the cosecant of the other.

cofunction of sine — cofunction

$$\sin x = \boxed{\cos(90° - x)} \; ; \; \cos x = \boxed{\sin(90° - x)}$$

$$\tan x = \cot(90° - x) \; ; \; \cot x = \tan(90° - x)$$

$$\sec x = \csc(90° - x) \; ; \; \csc x = \sec(90° - x)$$

Here x is an acute angle, i.e., $x < 90°$.

E.g. (a) $\cos 46° = \sin(90° - 46°) = \sin 44°$

Write the function in terms of its cofunction

(b) $\tan 71° = \cot(90° - 71°) = \cot 19°$

E.g. Assumption: all angles involved are acute.

① Solve for θ. Given that:

$$\cos(\theta + 4^\circ) = \sin(3\theta + 21^\circ)$$

This equation implies that $\theta + 4^\circ$ and $3\theta + 21^\circ$ are complementary angles. Hence,

$$(\theta + 4^\circ) + (3\theta + 21^\circ) = 90^\circ$$

$$4\theta + 25^\circ = 90^\circ$$

$$4\theta = 65^\circ$$

$$\theta = \frac{65^\circ}{4} = 16.25^\circ$$

② Solve for α. Given that

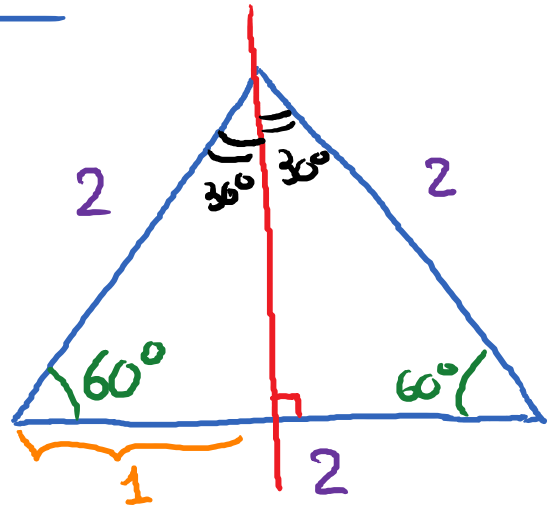
$$\tan(2\alpha - 18^\circ) = \cot(\alpha + 18^\circ)$$

Since tangent of one equals to cotangent of the other, they are complementary angle.

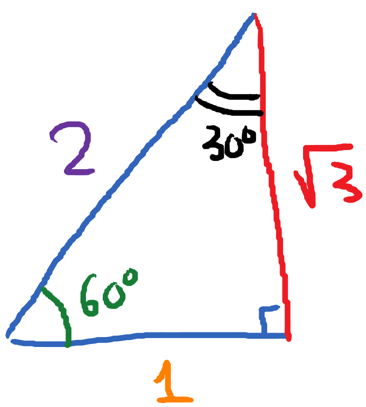
Hence, $2\alpha - 18^\circ + \alpha + 18^\circ = 90^\circ$

$$\alpha = 30^\circ$$

Obj 2: $30^\circ - 60^\circ$ Triangles.



Equilateral Triangle



$\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$; $\cot 60^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sec 60^\circ = 2$; $\csc 60^\circ = \frac{2}{\sqrt{3}}$

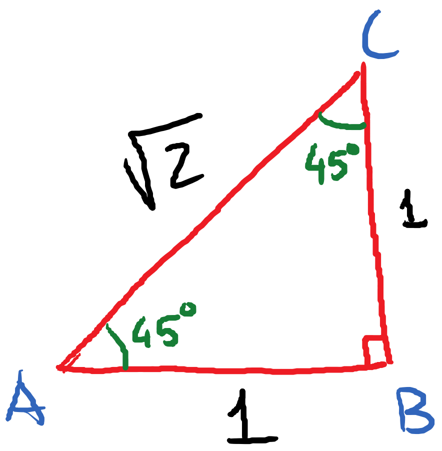
$= \frac{2\sqrt{3}}{3}$

$\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{\sqrt{3}}{3}$; $\cot 30^\circ = \sqrt{3}$

$\sec 30^\circ = \frac{2\sqrt{3}}{3}$; $\csc 30^\circ = 2$

Obj #3: 45° - 45° Right Triangle



$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

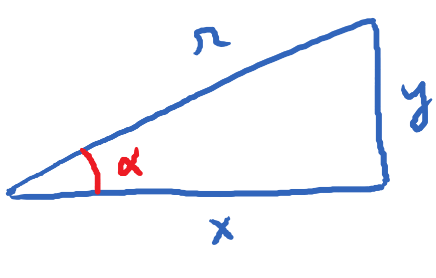
$\tan 45° = 1$

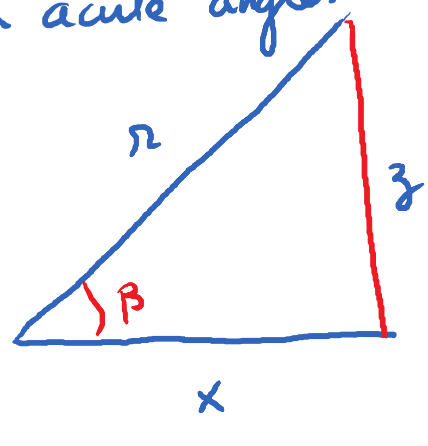
$\cot 45° = 1$

$\sec 45° = \sqrt{2}$

$\csc 45° = \sqrt{2}$

Obj #4: Increasing / Decreasing of trig functions. for acute angles





$\sin\alpha = \frac{y}{r}$ $\sin\beta = \frac{z}{r}$

$\sin\alpha < \sin\beta \longrightarrow$ the sine function is an increasing function csc is the reciprocal of sine

→ csc is a decreasing function.

Cos is a decreasing function

sec is an increasing function.

tan is an increasing

Cot is a decreasing