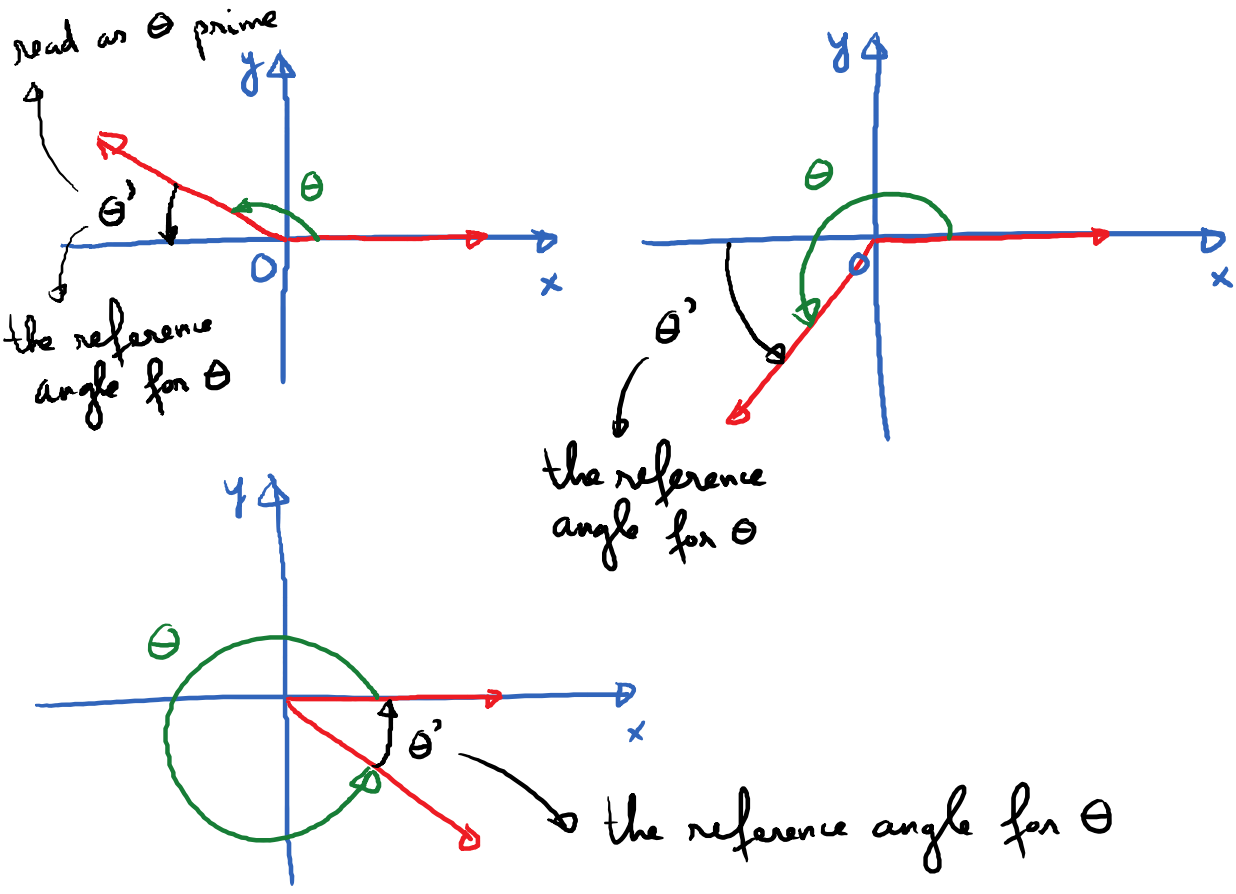
2.2 - Trig functions of non-acute angles

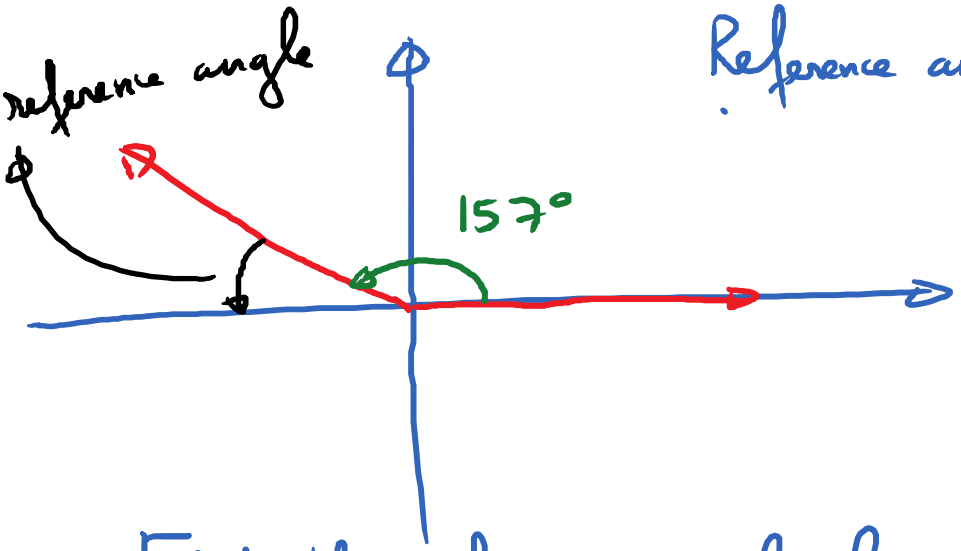
Wednesday, September 27, 2017 9:36 AM

Obj 1: Reference Angles.

A reference angle for a given angle θ is the positive acute angle made by the terminal side of θ with the x-axis.



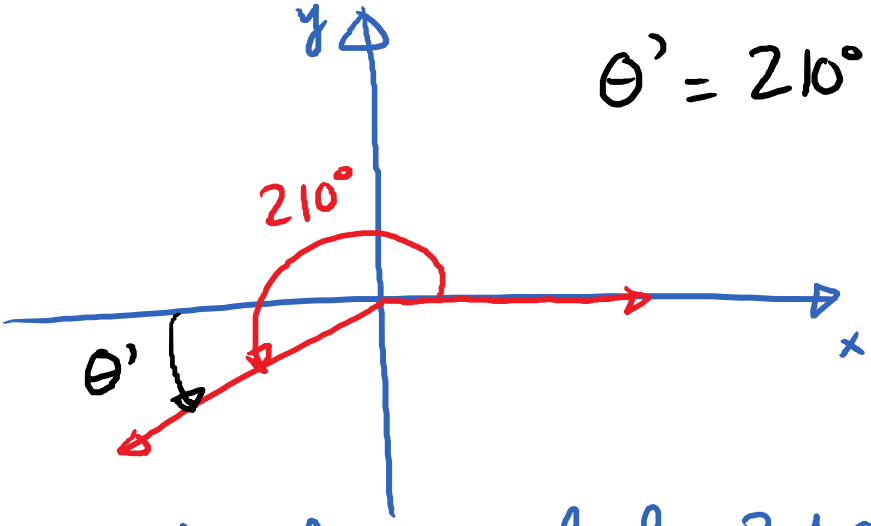
E.g. Find the reference angle for 157°.



Reference angle = 180° − 157°

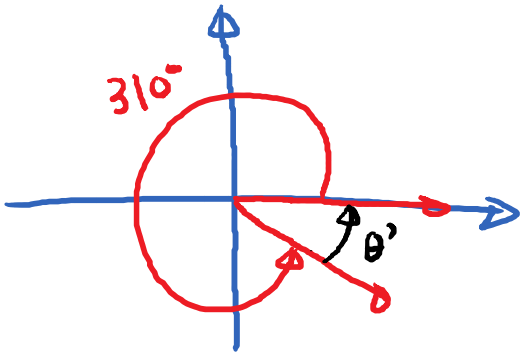
= 23°

E.g. Find the reference angle for 210°



$\theta' = 210° - 180° = 30°$

E.g. Find the reference angle for 310°



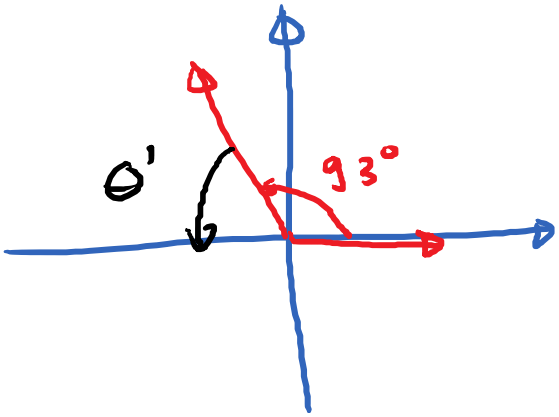
$\theta' = 360° - 310° = 50°$

E.g. Find the reference angle of 1893°

1st step: Find the least positive angle that is coterminal with this:

$$1893^\circ - 5 \cdot 360^\circ = 93^\circ$$

2nd: Find the reference angle for 93°



$$\theta' = 180^\circ - 93^\circ = \boxed{87^\circ}$$

E.g. Find the reference angle for -1776°

1st step: Find the least positive coterminal angle:

$$-1776^\circ + 5 \cdot 360^\circ = 24^\circ$$

2nd step: do nothing. Reference angle $= 24^\circ$.

E.g. Find reference angle for -1700°

1st step: Find least positive coterminal angle:

$$-1700^\circ + 5 \cdot 360^\circ = 100^\circ.$$

2nd step: Find reference angle:

$$180^\circ - 100^\circ = \boxed{80^\circ}$$

Obj #2: Use reference angle to find the trig values of angles that are related to one of those "nice" angles.

E.g. Find $\sin 675^\circ$, $\cos 675^\circ$, $\tan 675^\circ$.

1st step: Find the least positive angle coterminal with 675°

$$675^\circ - 360^\circ = 315^\circ.$$

2nd step: Find the reference angle of 315°

$$\theta' = 360^\circ - 315^\circ = 45^\circ.$$

3rd step: Find trig values of 45°

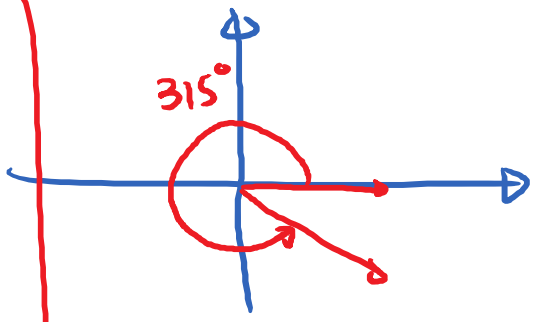
$\sin 45° = \frac{\sqrt{2}}{2}$; $\cos 45° = \frac{\sqrt{2}}{2}$; $\tan 45° = 1$

4th step:

$\sin 675° = -\frac{\sqrt{2}}{2}$

$\cos 675° = \frac{\sqrt{2}}{2}$

$\tan 675° = -1$



E.g. (a) Find $\cos 780°$, $\sin 780°$, $\tan 780°$

(b) Find $\sec(-405°)$, $\csc(-405°)$, $\cot(-405°)$

Sol (a) 1st step: Find the least positive coterminal angle with 780°.

$780° - 2 \cdot 360° = 60°$

Reference angle $= 60°$.

$\cos 60° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\tan 60° = \sqrt{3}$.

$\cos 780° = \frac{1}{2}$ $\sin 780° = \frac{\sqrt{3}}{2}$ $\tan 780° = \sqrt{3}$.

$\cos 780^\circ = \frac{1}{2}$ $\sin 780^\circ = \frac{\sqrt{3}}{2}$ $\tan 780^\circ = \sqrt{3}$.

(b) 1st step: Find the least positive coterminal angle with -405°

$$-405^\circ + 2 \cdot 360^\circ = 315^\circ$$

2nd step: Find reference angle: $360^\circ - 315^\circ = 45^\circ$

3rd step: $\sec 45^\circ = \sqrt{2}$; $\csc 45^\circ = \sqrt{2}$ $\cot 45^\circ = 1$

4th step: $\sec(-405^\circ) = \sqrt{2}$ $\csc(-405^\circ)$ $\cot(-405^\circ)$

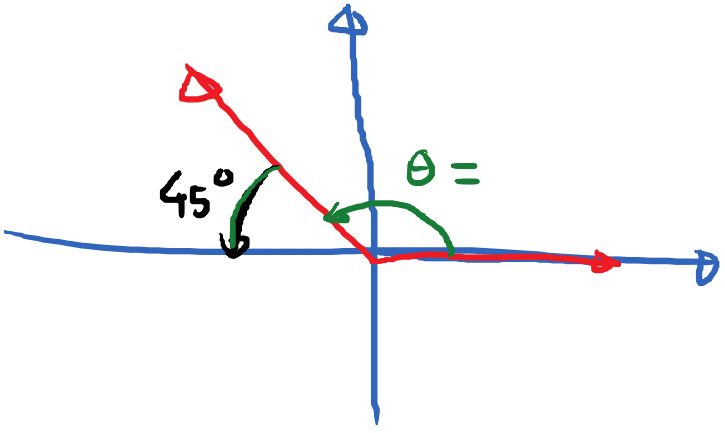
$= -\sqrt{2}$ $= -1$.

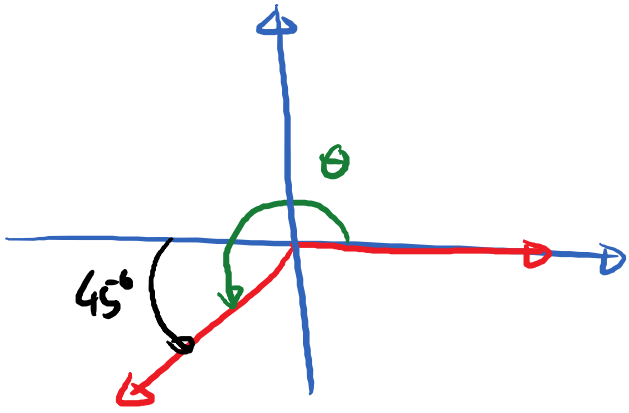
b/c -405° belongs to 4th quadrant.

E.g. Find all possible values of the angle θ in the interval $[0, 360^\circ)$ such that

$$\cos\theta = -\frac{\sqrt{2}}{2}.$$

Sol: Since $\cos\theta < 0$, θ is in II or III quadrant





Since the absolute value of $\cos\theta$ is $\frac{\sqrt{2}}{2}$, the reference angle for θ must be 45°

In first case: $\theta = 180^\circ - 45^\circ = \boxed{135^\circ}$

In second case: $\theta = 180^\circ + 45^\circ = \boxed{225^\circ}$

E.g. Evaluate

$$\cos 120^\circ + 2\sin^2 60^\circ - \tan^2 30^\circ$$

$$= -\cos 60^\circ + 2(\sin 60^\circ)^2 - (\tan 30^\circ)^2$$

$$= -\frac{1}{2} + 2\cdot\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= -\frac{1}{2} + 2\cdot\frac{3}{4} - \frac{1}{3}$$

$$= -\frac{1}{2} + \frac{3}{2} - \frac{1}{3} = \cdots$$