3.2. Application of Radian Measures.
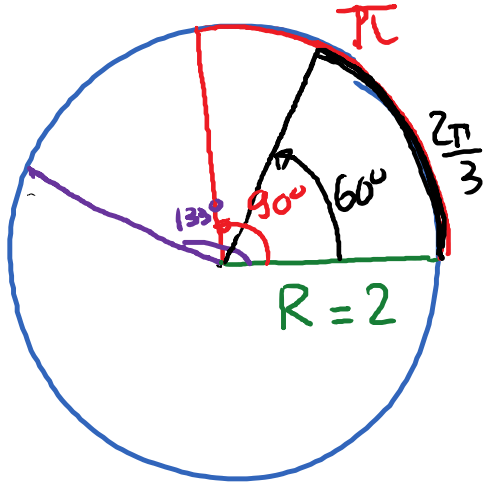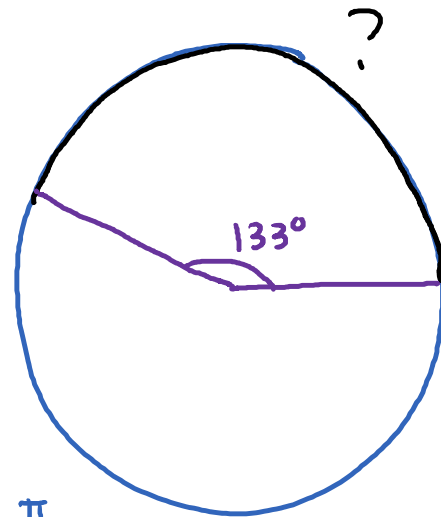
Obj 1: Arc length on a circle.

$$\text{Circumference} = 2\pi \cdot R = 2\pi \cdot 2 = 4\pi.$$

π

$\frac{2\pi}{3}$

$133°$ $90°$ $60°$

$R = 2$

?

$133°$

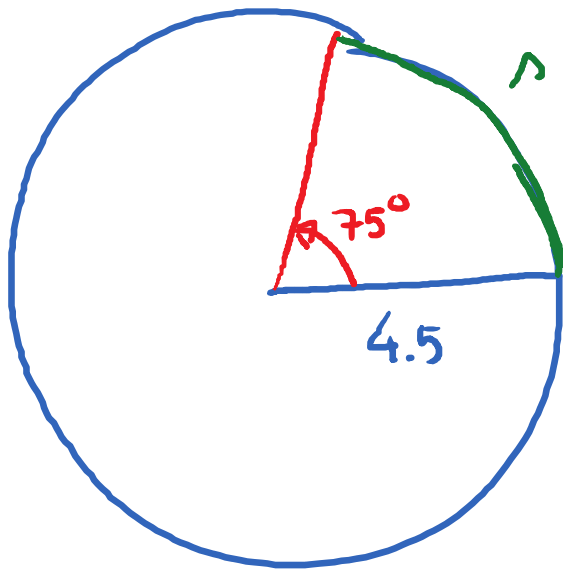Convert to radian: $133 \cdot \dfrac{\pi}{180} = \dfrac{133\pi}{180}$

radian measure.

Theorem: The length s of the arc subtended by an angle whose measure is x radian in a circle with radius R is given by:

$$\boxed{s = R \cdot x}$$

E.g.   Circle with radius $R = 4.5$.

Angle measure $= 75°$
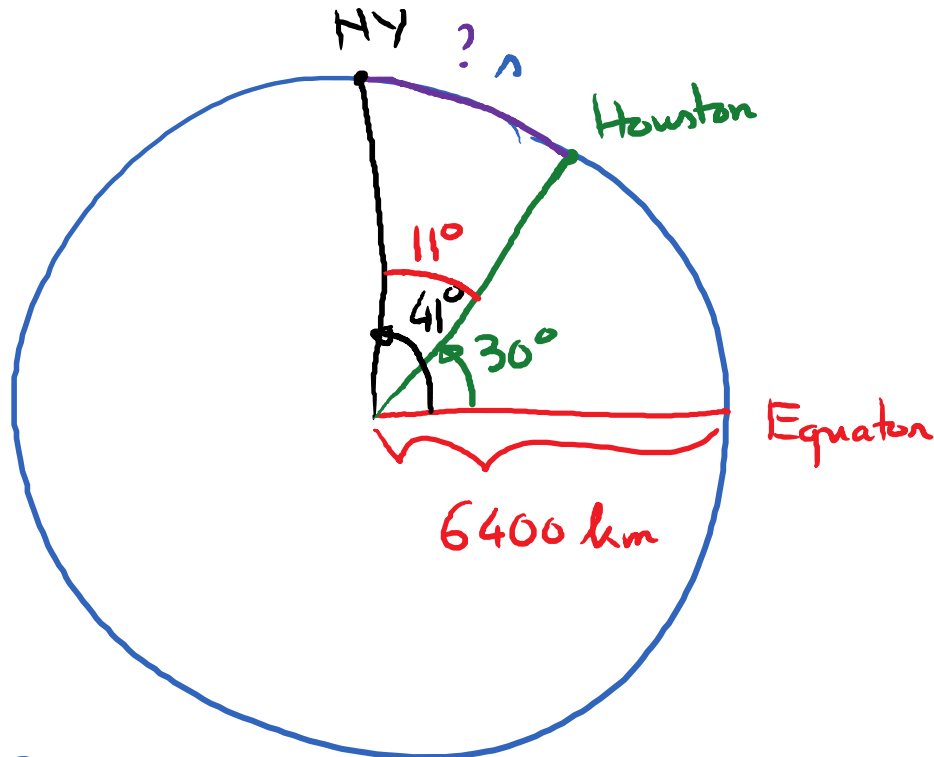
Find s.

(arc subtended by the $75°$ angle)

4.5

1st step: Convert $75°$ into radian.

$$75° = \left(75 \cdot \frac{\pi}{180} \right) \text{radians}$$
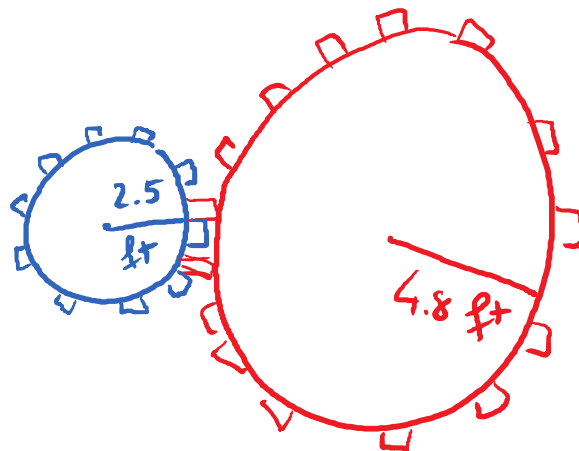
$$= \frac{15\pi}{36} = \frac{5\pi}{12} \text{ radians}.$$

2nd step: $s = \underbrace{(4.5)}_{R} \cdot \underbrace{\left(\frac{5\pi}{12} \right)}_{x} \approx \frac{22.5\pi}{12} \approx 5.89$

E.g.

NY ? ⌒

Houston

11°

41°

30°

Equator

6400 km

$$s \stackrel{?}{=} \left(11° \cdot \frac{\pi}{180} \right) \cdot (6400) \approx 1228.7 \; km$$

E.g.　2 gears

2.5 ft

4.8 ft

Q: If the smaller gear rotate through an angle of 225°. Q: Through how many degrees will the larger gear rotate?

Calculate the length that the smaller gear rotates:

$$\left(225° \cdot \frac{\pi}{180}\right) \cdot (2.5) = \frac{225\pi}{180} \cdot (2.5)$$

$$= \frac{5\pi}{4} \cdot (2.5) = \frac{12.5\pi}{4}$$

This is also the length that the larger gear $^{(ft)}$ rotates.

$$s = \frac{12.5\pi}{4}$$

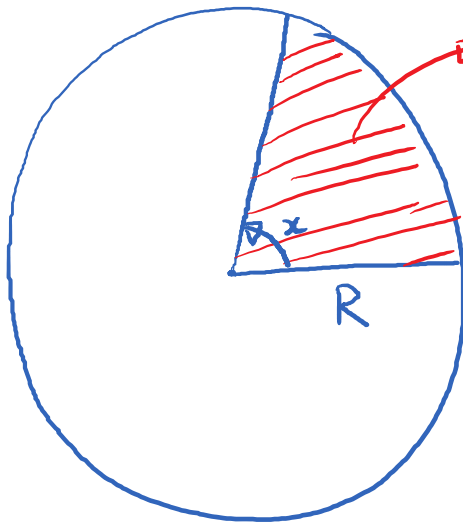Radius of larger gear $= 4.8$ ft

Angle of rotation for the larger gear is:

$$\frac{\frac{12.5\pi}{4}}{4.8} = \frac{12.5\pi}{4} \cdot \frac{1}{4.8}$$

$$= \left(\frac{12.5}{4} \cdot \frac{1}{4.8}\right)\pi$$

$$= 0.651\pi \quad \text{radian.}$$

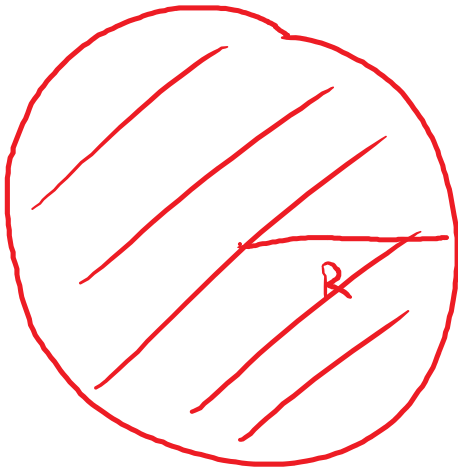Convert to degrees $0.651 \pi \cdot \dfrac{180}{\pi} = 117°.188$

Obj 2: Area of a a sector.

Area = ?

$Area = \dfrac{x \cdot R^2}{2}$
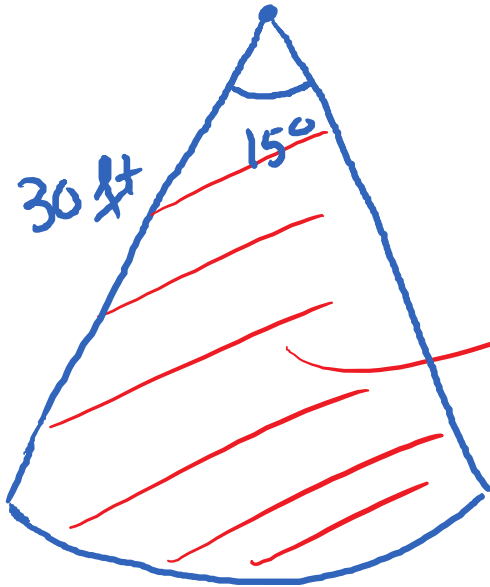
$= \dfrac{1}{2} R^2 \boxed{x}$

measured in radian

Area of whole circle

$= \pi R^2$

E.g. Irrigation System.

Convert $15°$ to radian:

$$15 \cdot \frac{\pi}{180} = \frac{15\pi}{180} = \frac{\pi}{12}.$$

radian

30 ft   15°

Area = ?

$$Area = \frac{1}{2} \cdot (30)^2 \cdot \frac{\pi}{12}$$

$$= \frac{450\pi}{12}$$

$$\approx 117.8 \; ft^2$$